First assignment of MP-206

Answers to questions 1-13 and 17-26

1) Normal stress along $\{2\ 1\ 2\}^T$: 32/9 MPa
Principal stresses: $\sigma_I = 4$ MPa, $\sigma_{II} = 1$ MPa, $\sigma_{III} = -2$ MPa

Principal directions: $\{n_I\} = \frac{1}{\sqrt{6}}\begin{Bmatrix} 2 \\ 1 \\ 1 \end{Bmatrix}$, $\{n_{II}\} = \frac{1}{\sqrt{3}}\begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix}$, $\{n_{III}\} = \frac{1}{\sqrt{2}}\begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix}$

2) Initially, suppose that $[\sigma]$ is a stress state described in the xyz reference system and $[\sigma']$ is the same stress state but described in another reference system $x'y'z'$. If $[l]$ is the transformation matrix then $[\sigma'] = [l]^T[\sigma][l]$. The transformation matrix has two important properties: $[l]^T[l] = [I]$ and $\det([l]) = \pm 1$. Therefore,

$\det([\sigma'] - \lambda[I]) = \det([l]^T[\sigma][l] - \lambda[l]^T[l]) = \det([l]^T([\sigma] - \lambda[I])[l])) = \det([l]^T)\det([\sigma] - \lambda[I])\det([l]) = \det([\sigma] - \lambda[I])$.

Thus, $[\sigma]$ and $[\sigma']$ have the same eigenvalues. Since the eigenvalues are also the roots of the polynomial equation $\lambda^3 - I_1\lambda^2 - I_2\lambda - I_3 = 0$, for them to be the same the polynomial equation $\lambda^3 - I_1\lambda^2 - I_2\lambda - I_3 = 0$ cannot change from one reference system to the other. It means that I_1, I_2, I_3 are invariants.

3) Let us say that $\alpha = \{x\}^T[A]_a\{x\}$ is a number. Hence α^T is also a number. Thus, $\alpha^T = (\{x\}^T[A]_a\{x\})^T = \{x\}^T([A]_a)^T\{x\} = -\{x\}^T[A]_a\{x\} = -\alpha$. Since $\alpha^T = \alpha$ one concludes that $\alpha = 0$.

4) $[\sigma]$ is symmetric and the eigenvalues of symmetric matrices are always real.

5) See solution of question 2.

6) If, in the xyz reference system $\sigma_x + \sigma_y + \sigma_z = 0$ holds, then we must show that there exists a $x'y'z'$ reference system such that $\sigma_{x'} = \sigma_{y'} = \sigma_{z'} = 0$. Since $\sigma_x + \sigma_y + \sigma_z = 0$, at least one of these normal stresses must be positive and at least one of them must be negative. Suppose $\sigma_x > 0$ and $\sigma_z < 0$.

Analysis of all planes parallel to σ_y, from the plane where σ_x is defined up to the plane where σ_z is defined, allows one to conclude that the normal stresses on these planes vary continuously from σ_x (positive value) to σ_z (negative value). Since the variation is continuous, one of the planes parallel to σ_y will yield zero normal stress. Let us call this plane β. Therefore, one can define three mutually orthogonal planes: one of them is the plane where σ_y is defined; another one is plane β; the third plane must have normal stress equal to $-\sigma_y$ since the invariant equation $\sigma_x + \sigma_y + \sigma_z = 0$ must hold.

If we consider all planes perpendicular to plane β, since the plane where $-\sigma_y$ is defined up to the plane where $+\sigma_y$ is defined, the normal stresses, vary, again continuously from a negative value ($-\sigma_y$) to a positive value ($+\sigma_y$). Since the variation is continuous, one of the planes perpendicular plane β yields zero normal stress. Let us call this plane γ. Therefore, one can define, again, three mutually orthogonal planes: one of them is the plane β; another one is plane γ; the third plane must have zero normal stress since the invariant equation $\sigma_x + \sigma_y + \sigma_z = 0$ must hold.

7) Deformation gradient: $[F]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 3/2\\ 0 & 0 & 3/2\end{bmatrix}$

$$x = X \quad , \quad y = 3+Y+3Z/2 \quad , \quad z = 3+3Z/2$$
$$u = x - X = 0 \quad , \quad v = y - Y = 3+3Z/2 \quad , \quad w = z - Z = 3+Z/2$$

8) Green strain tensor: $[\varepsilon]=\frac{1}{2}([F]^T[F]-[I])=\frac{1}{4}\begin{bmatrix}0 & 3 & 0\\ 3 & 7 & 0\\ 0 & 0 & 0\end{bmatrix}$

Small strain tensor: $\frac{1}{2}([H]^T+[H])=\frac{1}{4}\begin{bmatrix}0 & 3 & 0\\ 3 & 2 & 0\\ 0 & 0 & 0\end{bmatrix}$

9) At a certain instant of time the displacement field in a solid is given by

$$u = X^2 \quad , \quad v = YZ \quad , \quad w = 2XZ + X^2$$

Determine:

(a) The Green strain tensor.

(b) The strain at point $X = Y = 1$, $Z = 0$.

(c) The normal strain along direction $\{2\ 2\ 1\}^T$ at point $X = Y = 1$, $Z = 0$.

(d) The angle change between the perpendicular directions $\{2\ 2\ 1\}^T$ and $\{3\ 0\ -6\}^T$.

(e) The angle change between directions $\{2\ 2\ 1\}^T$ and $\{3\ 0\ 4\}^T$.

(a) $[H]=\begin{bmatrix}u_{,X} & u_{,Y} & u_{,Z}\\ v_{,X} & v_{,Y} & v_{,Z}\\ w_{,X} & w_{,Y} & w_{,Z}\end{bmatrix}=\begin{bmatrix}2X & 0 & 0\\ 0 & Z & Y\\ 2(X+Z) & 0 & 2X\end{bmatrix}$

$$[\varepsilon]=\frac{1}{2}([H]+[H]^T+[H]^T[H])=$$

$$\frac{1}{2}\begin{bmatrix}4(X+2Y^2+2XZ+Z^2) & 0 & 2(X+Z+2X^2+2XZ)\\ 0 & Z(2+Z) & Y(1+Z)\\ 2(X+Z+2X^2+2XZ) & Y(1+Z) & 4X+4X^2+Y^2\end{bmatrix}$$

(b) $[\varepsilon(1,1,0)] = \frac{1}{2}\begin{bmatrix} 12 & 0 & 6 \\ 0 & 0 & 1 \\ 6 & 1 & 9 \end{bmatrix}$

(c) $\{N\} = \{2\ 2\ 1\}^T/3 \Rightarrow$ $\varepsilon_n = \{N\}^T[\varepsilon]\{N\} = \frac{1}{18}\begin{Bmatrix} 2 \\ 2 \\ 1 \end{Bmatrix}^T \begin{bmatrix} 12 & 0 & 6 \\ 0 & 0 & 1 \\ 6 & 1 & 9 \end{bmatrix}\begin{Bmatrix} 2 \\ 2 \\ 1 \end{Bmatrix} = \frac{85}{18}$

$\{n\} = [F]\{N\} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}\frac{1}{3}\begin{Bmatrix} 2 \\ 2 \\ 1 \end{Bmatrix} = \frac{1}{3}\begin{Bmatrix} 6 \\ 3 \\ 7 \end{Bmatrix}$

(d) $\{S\} = \{3\ 0\ -6\}^T/3\sqrt{5} \Rightarrow$ $\{s\} = [F]\{S\} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}\frac{1}{3\sqrt{5}}\begin{Bmatrix} 3 \\ 0 \\ -6 \end{Bmatrix} = \frac{1}{\sqrt{5}}\begin{Bmatrix} 3 \\ -2 \\ -4 \end{Bmatrix}$

$\{n\}^T\{s\} = |n||s|\cos\theta \quad \Rightarrow \quad \theta = 107.84^o$

10) (a) $[F] = \begin{bmatrix} x_{,X} & x_{,Y} & x_{,Z} \\ y_{,X} & y_{,Y} & y_{,Z} \\ z_{,X} & z_{,Y} & z_{,Z} \end{bmatrix} = \begin{bmatrix} a_1 & 2a_1 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}$

$\{ab\} = [F]\{AB\} \quad \Rightarrow \quad \{ab\} = \begin{bmatrix} a_1 & 2a_1 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 3a_1 \\ a_2 \\ a_3 \end{Bmatrix} \quad \Rightarrow \quad |ab| = \sqrt{9a_1^2 + a_2^2 + a_3^2}$

(b) $\{ac\} = [F]\{AC\} \quad \Rightarrow \quad \{ac\} = \begin{bmatrix} a_1 & 2a_1 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 3a_1 \\ a_2 \\ 0 \end{Bmatrix} \quad \Rightarrow \quad |ac| = \sqrt{9a_1^2 + a_2^2}$

The angle θ_1 between $\{AB\}$ and $\{AC\}$ is computed through $\{AB\}^T\{AC\} = |AB||AC|\cos\theta_1$. Thus, $\cos\theta_1 = 2/\sqrt{6}$.

The angle θ_2 between $\{ab\}$ and $\{ac\}$ is computed through $\{ab\}^T\{ac\} = |ab||ac|\cos\theta_2$. Thus,

$$\frac{2}{\sqrt{6}} = \sqrt{\frac{9a_1^2 + a_2^2}{9a_1^2 + a_2^2 + a_3^2}} \quad \Rightarrow \quad 9a_1^2 + a_2^2 - 2a_3^2 = 0$$

11) Simply check that the invariants are different.

12)

$$[C]=\begin{bmatrix}157.85 & 5.705 & 5.705 & 0 & 0 & 0\\ 5.705 & 15.55 & 7.266 & 0 & 0 & 0\\ 5.705 & 7.266 & 15.55 & 0 & 0 & 0\\ 0 & 0 & 0 & 3.2 & 0 & 0\\ 0 & 0 & 0 & 0 & 4.4 & 0\\ 0 & 0 & 0 & 0 & 0 & 4.4\end{bmatrix}\text{GPa}$$

13) Suppose the stress tensor $[\sigma]$ is available and the associated eigenvalue problem is $([\sigma] - \lambda[I])\{n\} = \{0\}$ where $\{n\} = \{n_x\ n_y\ n_z\}^T$. Therefore,

$$\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{bmatrix}\begin{Bmatrix}n_x\\ n_y\\ n_z\end{Bmatrix}=\lambda\begin{Bmatrix}n_x\\ n_y\\ n_z\end{Bmatrix} \quad\Rightarrow\quad \begin{matrix}\sigma_x n_x+\tau_{xy}n_y+\tau_{xz}n_z=\lambda n_x\\ \tau_{xy}n_x+\sigma_y n_y+\tau_{yz}n_z=\lambda n_y\\ \tau_{xz}n_x+\tau_{yz}n_y+\sigma_z n_z=\lambda n_z\end{matrix}$$

The constitutive relation for a completely isotropic material reads:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \varepsilon_{yz}\\ \varepsilon_{xz}\\ \varepsilon_{xy}\end{Bmatrix}=\begin{bmatrix}S_{11} & S_{12} & S_{12} & 0 & 0 & 0\\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0\\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & S_{11}-S_{12} & 0 & 0\\ 0 & 0 & 0 & 0 & S_{11}-S_{12} & 0\\ 0 & 0 & 0 & 0 & 0 & S_{11}-S_{12}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{Bmatrix}S_{11}\sigma_x+S_{12}(\sigma_y+\sigma_z)\\ S_{11}\sigma_y+S_{12}(\sigma_x+\sigma_z)\\ S_{11}\sigma_z+S_{12}(\sigma_x+\sigma_y)\\ (S_{11}-S_{12})\tau_{yz}\\ (S_{11}-S_{12})\tau_{xz}\\ (S_{11}-S_{12})\tau_{xy}\end{Bmatrix}$$

The strain tensor $[\varepsilon]$ multiplied by vector $\{n\}$ can be written as

$$\begin{bmatrix}\varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z\end{bmatrix}\begin{Bmatrix}n_x\\ n_y\\ n_z\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x n_x+\varepsilon_{xy}n_y+\varepsilon_{xz}n_z\\ \varepsilon_{xy}n_x+\varepsilon_y n_y+\varepsilon_{yz}n_z\\ \varepsilon_{xz}n_x+\varepsilon_{yz}n_y+\varepsilon_z n_z\end{Bmatrix}$$

The first row of $[\varepsilon]\{n\}$ is given by

$$\begin{aligned}&\varepsilon_x n_x+\varepsilon_{xy}n_y+\varepsilon_{xz}n_z=\\ &[S_{11}\sigma_x+S_{12}(\sigma_y+\sigma_z)]n_x+(S_{11}-S_{12})(\tau_{xy}n_y+\tau_{xz}n_z)=\\ &S_{11}(\sigma_x n_x+\tau_{xy}n_y+\tau_{xz}n_z)+S_{12}[(\sigma_y+\sigma_z)n_x-\tau_{xy}n_y-\tau_{xz}n_z]\end{aligned}$$

If equation $\sigma_x n_x + \tau_{xy} n_y + \tau_{xz} n_z = \lambda n_x$ is used the first row of $[\varepsilon]\{n\}$ simplifies to

$$\varepsilon_x n_x + \varepsilon_{xy} n_y + \varepsilon_{xz} n_z = [S_{11}\lambda + S_{12}(\sigma_x + \sigma_y + \sigma_z - \lambda)]n_x$$

Similarly, the second and third rows of $[\varepsilon]\{n\}$ simplify to

$$\varepsilon_{xy} n_x + \varepsilon_y n_y + \varepsilon_{yz} n_z = [S_{11}\lambda + S_{12}(\sigma_x + \sigma_y + \sigma_z - \lambda)]n_y$$
$$\varepsilon_{xz} n_x + \varepsilon_{yz} n_y + \varepsilon_z n_z = [S_{11}\lambda + S_{12}(\sigma_x + \sigma_y + \sigma_z - \lambda)]n_z$$

Thus,

$$\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_y & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_z \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = [S_{11}\lambda + S_{12}(\sigma_x + \sigma_y + \sigma_z - \lambda)] \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

proving that the principal stress directions coincide with the principal strain directions.

17) G_{12} modulus:

$$G_{12} = \left[\frac{4}{(E_x)_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + 2\frac{\nu_{12}}{E_1} \right]^{-1}$$

Approximation for $E_1 >> E_2$

$$G_{12} \approx \left[\frac{4}{(E_x)_{45}} - \frac{1}{E_2} + 2\frac{\nu_{21}}{E_2} \right]^{-1}$$

18) $$(\nu_{xy})_{45} = \frac{\dfrac{2\nu_{21}}{E_2} - \dfrac{1}{E_1} - \dfrac{1}{E_2} + \dfrac{1}{G_{12}}}{\dfrac{1}{E_1} + \dfrac{1}{E_2} + \dfrac{1}{G_{12}} - \dfrac{2\nu_{21}}{E_2}} \approx \frac{\dfrac{2\nu_{21}}{E_2} - \dfrac{1}{E_2} + \dfrac{1}{G_{12}}}{\dfrac{1}{E_2} + \dfrac{1}{G_{12}} - \dfrac{2\nu_{21}}{E_2}}$$

19) $(E_x)_{30} = E_2$ 32/19 and $(\nu_{xy})_{30}/ (E_x)_{30} = 17/(96E_2)$. Thus, $(\nu_{xy})_{30} = 17/57 = 0.298246$.

20)

$$\frac{E_2}{(E_x)_{30}} = \frac{9E_2}{16E_1} + \frac{1}{16} + \frac{3}{16}\left(\frac{E_2}{G_{12}} - 2\nu_{12}\frac{E_2}{E_1} \right) \quad , \quad \frac{E_2}{(E_x)_{60}} = \frac{E_2}{16E_1} + \frac{9}{16} + \frac{3}{16}\left(\frac{E_2}{G_{12}} - 2\nu_{12}\frac{E_2}{E_1} \right)$$

Combining the two relations above, $\dfrac{1}{(E_x)_{30}} + \dfrac{1}{2E_2} = \dfrac{1}{(E_x)_{60}} + \dfrac{1}{2E_1}$

21) E_x modulus:

$$E_x = \left[\frac{1}{E_1} c^4 + \frac{1}{E_2} s^4 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) s^2 c^2 \right]^{-1}$$

First derivative:

$$\frac{dE_x}{d\theta} = -E_x^2 \left[-\frac{4}{E_1} c^3 s + \frac{4}{E_2} cs^3 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) (-2cs^3 + 2c^3 s) \right] = 0$$

$$\frac{dE_x}{d\theta} = -2E_x^2 cs \left[c^2 \left(\frac{1}{G_{12}} - \frac{2(1+\nu_{12})}{E_1} \right) + s^2 \left(-\frac{1}{G_{12}} + \frac{2}{E_2} + \frac{2\nu_{12}}{E_1} \right) \right] = 0$$

Let $A = -\frac{1}{G_{12}} + \frac{2(1+\nu_{12})}{E_1}$, $B = -\frac{1}{G_{12}} + \frac{2}{E_2} + \frac{2\nu_{12}}{E_1} = -\frac{1}{G_{12}} + \frac{2(E_1/E_2 + \nu_{12})}{E_1}$

then

$$\frac{dE_x}{d\theta} = -2E_x^2 cs(-Ac^2 + Bs^2) = 0$$

The first derivative is zero when $\theta = 0^o$ ($s = 0$) and $\theta = 90^o$ ($c = 0$). In the range $0^o < \theta < 90^o$ the term $-c^2A + s^2B$ is zero only if A and B have the same sign. Usually $E_1 > E_2$. Thus, $B \geq A$. Second derivative:

$$\frac{d^2 E_x}{d\theta^2} = -2\frac{d}{d\theta}(E_x^2 cs)(-Ac^2 + Bs^2) - 4E_x^2 c^2 s^2 (A+B) = 0$$

i) If $A > 0$, then $B \geq A > 0$.

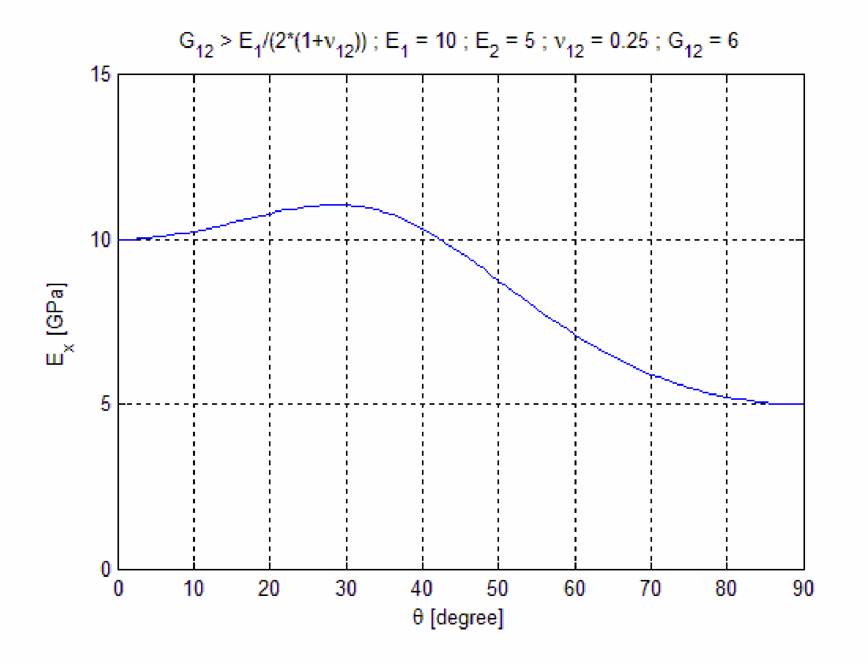
$$A = -\frac{1}{G_{12}} + \frac{2(1+\nu_{12})}{E_1} > 0 \quad \Rightarrow \quad G_{12} > \frac{E_1}{2(1+\nu_{12})}$$

ii) For $(-Ac^2 + Bs^2) = 0$ and $A > 0$, $B > 0$

$$\frac{d^2 E_x}{d\theta^2} = -2\frac{d}{d\theta}(E_x^2 cs)(-Ac^2 + Bs^2) - 4E_x^2 c^2 s^2 (A+B) = -4E_x^2 c^2 s^2 (A+B) < 0$$

Concavity is negative. Hence, it is a maximum.

Example: E_1 = 10 GPa, E_2 = 5 GPa, ν_{12} = 0.25, G_{12} = 6 GPa $\Rightarrow$ $G_{12} > E_1/2(1 + \nu_{12})$ = 4 GPa.



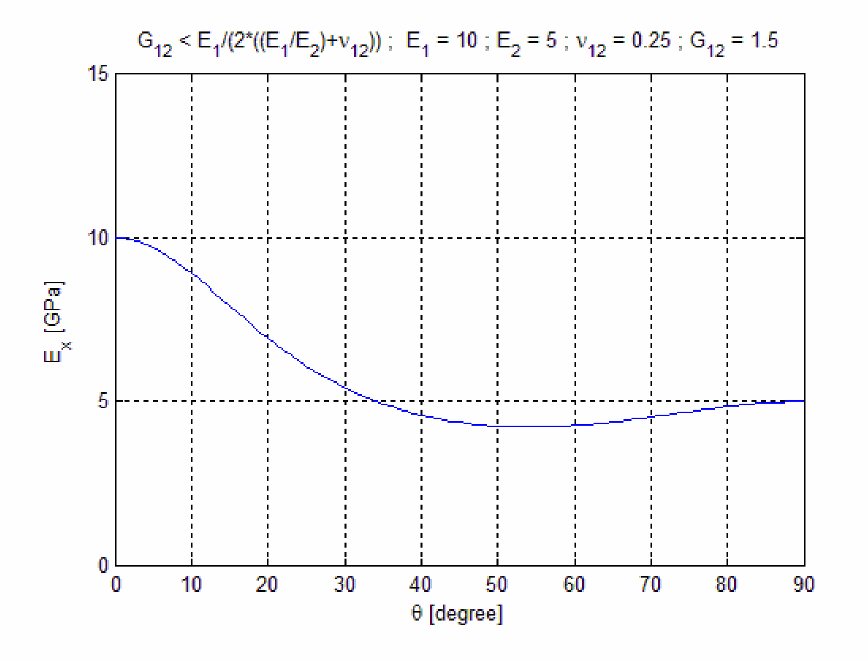
22)
i) If $B < 0$ then $A \leq B < 0$.

$$B = -\frac{1}{G_{12}} + \frac{2(E_1/E_2 + \nu_{12})}{E_1} < 0 \quad \Rightarrow \quad G_{12} < \frac{E_1}{2(E_1/E_2 + \nu_{12})}$$

ii) For $(-Ac^2 + Bs^2) = 0$ and $A < 0$, $B < 0$

$$\frac{d^2E_x}{d\theta^2} = -2\frac{d}{d\theta}(E_x^2 cs)(-Ac^2 + Bs^2) - 4E_x^2c^2s^2(A+B) = -4E_x^2c^2s^2(A+B) > 0$$

Example: E_1 = 10 GPa, E_2 = 5 GPa, ν_{12} = 0.25, G_{12} = 1.5 GPa $\Rightarrow$ $G_{12} < E_1/2(E_1/E_2 + \nu_{12})$ = 2.22 GPa.



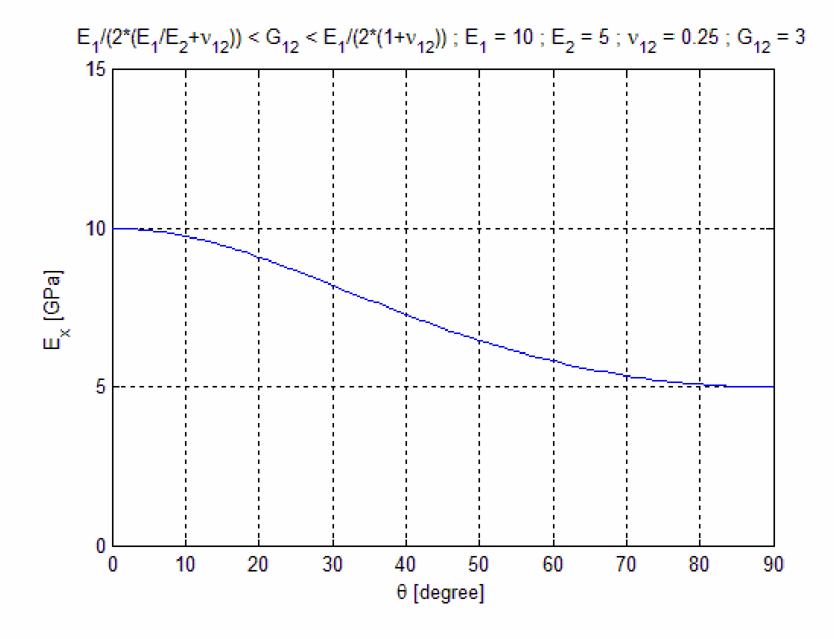
23)

i) If $A < 0$ and $B > 0$ then $(-Ac^2 + Bs^2) \neq 0$. In this case there are no extreme points in the range $0^o < \theta < 90^o$.

$$A = -\frac{1}{G_{12}} + \frac{2(1+\nu_{12})}{E_1} < 0 \quad \Rightarrow \quad G_{12} < \frac{E_1}{2(1+\nu_{12})}$$

$$B = -\frac{1}{G_{12}} + \frac{2(E_1/E_2+\nu_{12})}{E_1} > 0 \quad \Rightarrow \quad G_{12} > \frac{E_1}{2(E_1/E_2+\nu_{12})}$$

Example: E_1 = 10 GPa, E_2 = 5 GPa, ν_{12} = 0.25, G_{12} = 3 GPa. Then, $G_{12} < E_1/2(1 + \nu_{12})$ = 4 GPa and $G_{12} > E_1/2(E_1/E_2 + \nu_{12})$ = 2.22 GPa



Most typical composites have a minimum value for E_x in the range $0^o < \theta < 90^o$. In the case of unidirectional laminae the minimum value is not significantly different from E_2.

24)

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_0 \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \varepsilon_1 = \frac{\sigma_0}{E_1} \quad , \quad \varepsilon_2 = -\frac{\nu_{12}\sigma_0}{E_1}$$

$$\begin{Bmatrix} \varepsilon_1' \\ \varepsilon_2' \\ \gamma_{12}' \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_0 \\ \sigma_0 \\ 0 \end{Bmatrix} \Rightarrow \varepsilon_2' = \frac{-\nu_{12}\sigma_0}{E_1} + \frac{\sigma_0}{E_2} = \varepsilon_2 + k\varepsilon_1$$

25) $\varepsilon_{x45} = \frac{\tau_0}{2}\left(\frac{1}{E_1} - \frac{1}{E_2}\right) \quad , \quad \varepsilon_{y45} = \varepsilon_{x45} \quad , \quad \gamma_{x45} = \tau_0\left(\frac{1+\nu_{12}}{E_1} + \frac{1}{E_2}\right)$

26) $k = -S_{xs}/S_{ys}$