First assignment of MP-206

Resolution of this assignment is voluntary

1) The stress state at a point is

$$[\sigma] = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \text{MPa}$$

Obtain the normal stress along the direction $\{2\ 1\ 2\}^T$. Determine the principal stresses and their respective directions.

2) The condition $\det([\sigma] - \lambda[I]) = 0$ must be imposed in order to find the principal stresses. Algebraically, this condition leads to the polynomial equation $\lambda^3 - I_1\lambda^2 - I_2\lambda - I_3 = 0$, where I_1, I_2 and I_3 are the invariants of the stress state. Show that the invariants do not change under any transformation of the reference system.

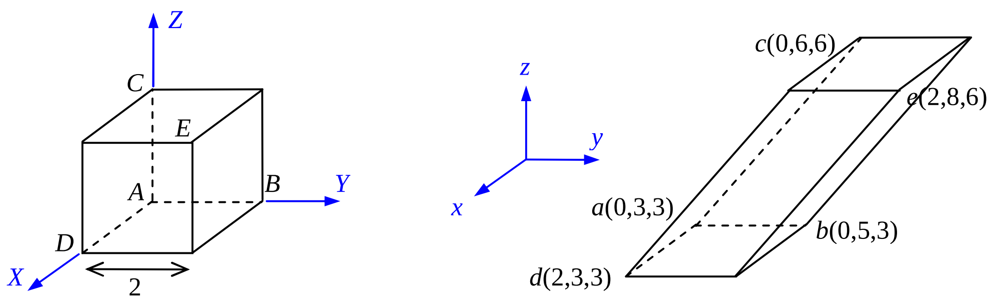
3) A square matrix $[A]$ can always be decomposed into symmetric and antisymmetric terms such that $[A] = [A]_s + [A]_a$, where $[A]_s = ([A] + [A]^T)/2$ and $[A]_a = ([A] - [A]^T)/2$. Show that the contribution of $[A]_a$ to $\{x\}^T[A]\{x\}$ is null, i.e., show that $\{x\}^T[A]_a\{x\} = 0$.

4) Show that the three roots of $\det([\sigma] - \lambda[I]) = 0$ must necessarily be real.

5) Consider that a given stress state $[\sigma]$ is described in another reference system as $[\sigma']$. Show that the eigenvalues of $[\sigma]$ and $[\sigma']$ must be the same.

6) Show that, if a stress state satisfies $\sigma_x + \sigma_y + \sigma_z = 0$, then it corresponds to a pure shear stress state. In other words, show that there exists a reference system $x'y'z'$ such that $\sigma_{x'} = \sigma_{y'} = \sigma_{z'} = 0$ provided $\sigma_x + \sigma_y + \sigma_z = 0$ holds.

7) A homogeneous transformation changes the configuration of the cube as show in the figure below. Fibers parallel to X remain parallel to X and do not stretch. Determine the deformation gradient, the transformations $x = x(X, Y, Z)$, $y = y(X, Y, Z)$, $z = z(X, Y, Z)$ and the displacement field $u = u(X, Y, Z)$, $v = v(X, Y, Z)$, $w = w(X, Y, Z)$.



8) Determine the Green strain tensor and the small strain tensor for the solid of problem 7.

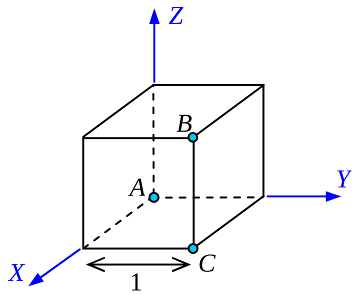
9) At a certain instant of time the displacement field in a solid is given by

$$u = X^2 \quad , \quad v = YZ \quad , \quad w = 2XZ + X^2$$

Determine:
(a) The Green strain tensor.
(b) The strain at point $X = Y = 1$, $Z = 0$.
(c) The normal strain along direction $\{2\ 2\ 1\}^T$ at point $X = Y = 1$, $Z = 0$.
(d) The angle change between the perpendicular directions $\{2\ 2\ 1\}^T$ and $\{3\ 0\ -6\}^T$.
(e) The angle change between directions $\{2\ 2\ 1\}^T$ and $\{3\ 0\ 4\}^T$.

10) A cube with unit length edges undergoes the transformation $x = a_1(X + 2Y)$, $y = a_2Y$, $z = a_3Z$, where a_1, a_2, a_3 are constants.

Determine:
(a) the length of the diagonal AB after deformation;
(b) The relation among a_1, a_2, a_3 such that there is no change in the angle between AB and AC.



11) Show that the two strain states below, referred to different coordinate systems, cannot represent the same strain state

$$[\varepsilon] = \begin{bmatrix} 5 & 3 & -1 \\ 3 & 4 & 0 \\ -1 & 0 & 2 \end{bmatrix} \quad , \quad [\varepsilon'] = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 6 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

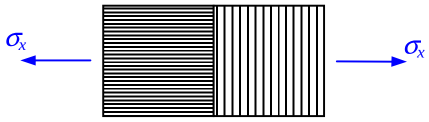
12) A graphite-epoxy orthotropic material is defined by E_1 = 155 GPa, $E_2 = E_3$ = 12.1 GPa, $G_{12} = G_{13}$ = 4.4 GPa, G_{23} = 3.2 GPa, $\nu_{12} = \nu_{13}$ = 0.25 and ν_{23} = 0.46. Write the constitutive matrix $[C]$.

13) Show that the principal stress directions coincide with the principal strain directions for a linear elastic isotropic material.

14) Obtain the stresses in an aluminum alloy with E = 72 GPa and G = 27 GPa knowing that the strains are $\varepsilon_1 = 200\times10^{-6}$, $\varepsilon_2 = 300\times10^{-6}$, $\varepsilon_3 = 0$, $\varepsilon_{12} = 100\times10^{-6}$, $\varepsilon_{13} = 0$ and $\varepsilon_{23} = 400\times10^{-6}$.

15) Using the equations of lamina stiffness transformation, prove that the quantity $Q_{11} + Q_{22} + 2Q_{12}$ is invariant, i.e., it is independent of axes orientation, by showing that, for any angle θ, the following relation holds: $\overline{Q}_{11} + \overline{Q}_{12} + 2\overline{Q}_{12} = Q_{11} + Q_{12} + 2Q_{12}$.

16) A specimen is made by joining along one edge two unidirectional laminae at 0^o and 90^o to the loading axis, as shown below. Compute the strains for σ_x = 100 MPa. Assume E_1 = 145 GPa, E_2 = 10.4 GPa, G_{12} = 7.6 GPa and ν_{12} = 0.28.



17) Knowing E_1, E_2, ν_{12} and $(E_x)_{45}$ (modulus at 45^o to the fiber direction), determine G_{12} exactly. Find an approximation for G_{12} using high stiffness composites.

18) Knowing E_1, E_2, G_{12} and ν_{12} for a unidirectional lamina determine $(\nu_{xy})_{45}$. Find an approximation for $(\nu_{xy})_{45}$ using high stiffness composites.

19) Determine Poisson's ratio ν_{xy} at angle $\theta = 30^o$ with the fiber direction for material with the following properties: $E_1/E_2 = 3$, $G_{12}/E_2 = 0.5$ and $\nu_{12} = 0.25$.

20) A unidirectional lamina is loaded at angles $\theta = 30^o$ and 60^o with the fiber direction and the corresponding moduli $(E_x)_{30}$ and $(E_x)_{60}$ are obtained. Determine a relationship between those two moduli and E_1 and E_2.

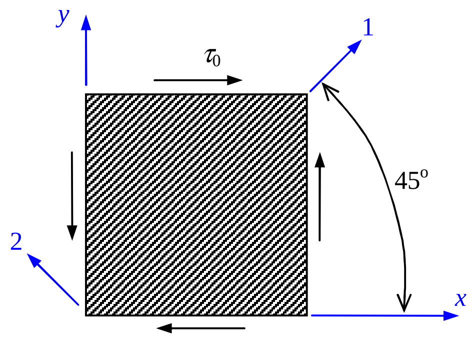
21) Using the transformation relation for E_x, determine its maximum and minimum values (by using derivatives of E_x with respect to θ). Prove that E_x can have a maximum for some value of θ ($0^o < \theta < 90^o$) when $2G_{12} > E_1/(1 + \nu_{12})$. This means that for some orthotropic materials, the off-axis modulus E_x can be higher than E_1.

22) Using the same procedure as in problem 22, prove that E_x can have a minimum for some value of θ ($0^o < \theta < 90^o$) when $2G_{12} < E_1/[(E_1/E_2) + \nu_{12}]$.

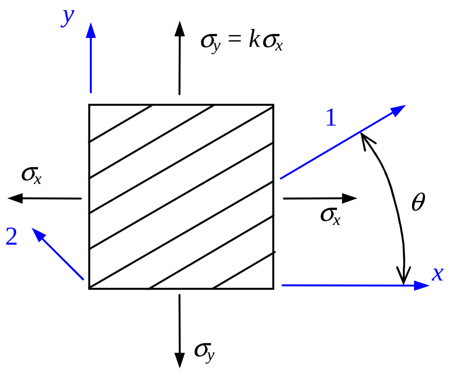
23) Using the same procedure as in problem 22, prove that E_x attains its maximum value E_1 at $\theta = 0^o$ and its minimum value E_2 at $\theta = 90^o$ when $E_1/[(E_1/E_2) + \nu_{12}] < 2G_{12} < E_1/(1 + \nu_{12})$.

24) A unidirectional lamina is loaded under a uniaxial stress $\sigma_1 = \sigma_0$ and the principal strains ε_1 and ε_2 are measured. Compute the transverse strain ε_2' of the same lamina loaded under equal biaxial normal stresses $\sigma_1 = \sigma_2 = \sigma_0$ as a function of ε_1, ε_2 and the ratio $k = E_1/E_2$.

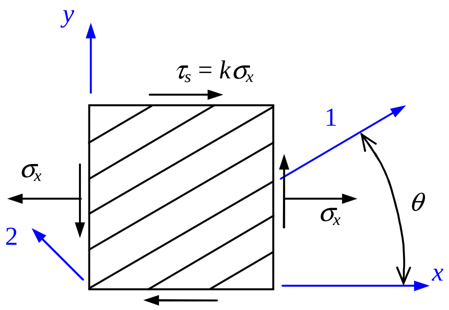
25) For a unidirectional lamina in pure shear τ_0 at 45^o with the fiber direction, obtain the expressions for the three strain components ε_x, ε_y and γ_{xy} as a function of the basic engineering constants E_1, E_2, G_{12}, ν_{12}, and the shear stress τ_0.



26) An off-axis unidirectional lamina is loaded under biaxial normal loading along the x- and y-axes. Find as expression for the ratio of the two normal stresses, $k = \sigma_y/\sigma_x$, such that there is no shear deformation in the lamina.



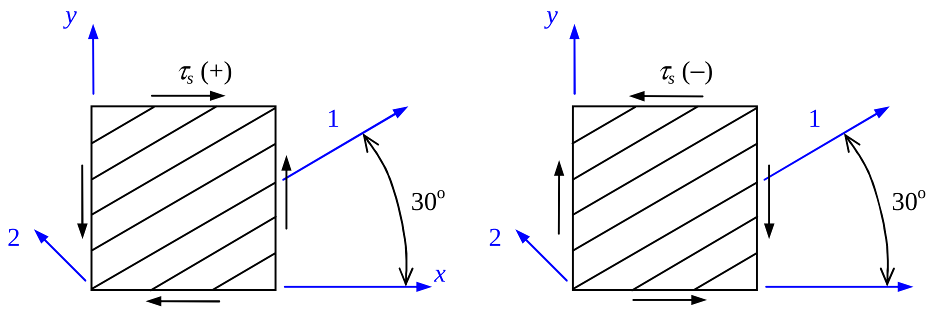
27) An off-axis unidirectional lamina is loaded as shown under uniaxial tension and in-plane shear at an angle to the fiber direction. Express the normal strains ε_x and ε_y in terms of the engineering properties (E_x, E_y, G_{xy}, ν_{xy}, ν_{yx}, η_{xs}, η_{ys}, η_{sx}, η_{sy}). What relation must the engineering properties satisfy, so that for a certain ratio σ_x/τ_s the material behaves as an infinitely rigid one with $\varepsilon_x = \varepsilon_y = 0$? What is the ratio $k = \tau_s/\sigma_x$ in that case?



28) A unidirectional lamina is loaded under biaxial normal loading $\sigma_x = -2\sigma_y = 2\sigma_0$ at 45° with the fiber direction. The basic strength properties of the material are: $X_t = X_c = 3Y_c = 5S = 12Y_t = 600$ MPa. Determine the level σ_x at failure of the lamina according to the maximum stress theory. What is the failure mode?

29) A unidirectional S-glass/epoxy lamina is loaded at an angle to the fiber direction. Using the maximum strain criterion, determine the off-axis strength, F_{xt}, and the fiber orientation θ at which the predictions of in-plane shear and transverse failure coincide. X_t = 1280 MPa, Y_t = 49 MPa, S = 69 MPa, ν_{12} = 0.27, ν_{21} = 0.06.

30) For the off-axis lamina under positive and negative shear stress as shown, and using the maximum strain failure theory, express the positive and negative shear strengths, S_+ and S_-, in terms of the basic lamina strengths (X_t, X_c, Y_c, Y_t, S) and material Poisson's ratios.



31) A unidirectional lamina is loaded under equal biaxial compression $\sigma_x = \sigma_y = -\sigma_0$ at 30° and −60° with the fiber direction. Calculate the ultimate value of σ_0 using the maximum strain theory for the following properties: $S = Y_t$ = 55 MPa, E_1 = 145 GPa, E_2 = 10.4 GPa, G_{12} = 6.9 GPa, ν_{12} = 0.27, X_c = 1725 MPa, Y_c = 207 MPa.

32) Express the Tsai-Hill failure criterion for pure shear loading of a lamina at an angle θ with the principal material axes and find an expression for the shear stress at failure τ_s in terms of X, Y and S.

33) A unidirectional lamina is loaded in pure shear τ_s at an angle θ = 30° with the fiber direction. Determine the shear stress at failure τ_s using the Tsai-Hill failure criterion and the following data: X = 2070 MPA, Y = 228 MPa, S = 69 MPa.

34) Using the Tsai-Hill failure criterion, determine the strength F_0 of a lamina under equal biaxial tension and shear ($\sigma_x = \sigma_y = 2\tau_s = F_0$) at 45° with the fiber direction in terms of X and Y.

35) An off-axis lamina is loaded under $\sigma_x = -\sigma_y = F_0$. Determine F_0 at failure using the Tsai-Hill and maximum failure criteria for a material of the following properties: X_t = 2280 MPa, Y_t = 59 MPa, S = 69 MPa, X_c = 1450 MPa, Y_c = 228 MPa.

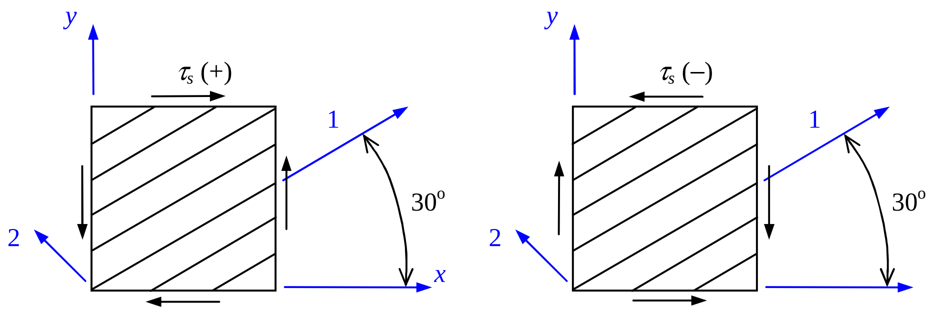
36) The off-axis strength of a unidirectional lamina can be higher than X at some angle between 0° and 90°. Using the Tsai-Hill failure criterion find a relationship among X, Y and S such that $X_\theta > X$ for some angle $0° < \theta < 90°$.

37) For the same conditions of problem 37 find a relationship among X, Y and S such that $X_\theta < Y$ for some angle $0° < \theta < 90°$.

38) Using the Tsai-Wu failure criterion, determine the strength of a lamina under the loading $-\sigma_x = \sigma_y = F_0$. Obtain the exact solution for the ultimate value of F_0 in terms of the Tsai-Wu coefficients F_1, F_2, F_{11}, etc. Use $\theta = 15^o$.

39) Using the Tsai-Wu failure criterion for pure shear loading of a lamina at an angle θ with the fiber direction, express the shear stress at failure $\tau_s = F_s$ in terms of the Tsai-Wu coefficients. Particularize the expression obtained for $\theta = 45^o$.

40) For the off-axis lamina under positive and negative shear stress as shown, express the positive and negative shear strengths, S_+ and S_-, using the Tsai-Wu failure theory in terms of the polynomial coefficients (F_1, F_2, F_{11}, F_{22}, F_{12}).



41) For the off-axis lamina of problem 41, obtain expressions for the coefficient F_{12} of the Tsai-Wu criterion in terms of the basic strength parameters and positive or negative shear strengths S_+ and S_-. Compare the values of F_{12} based on S_+ and S_- by assuming $F_1 << F_2$, $F_{11} << F_{22}$.