Third assignment of MP-206

Resolution of this assignment is voluntary

1) A laminate of 0.015 in thickness under a complex load gives the following midplane strains and curvatures: $\varepsilon_{x0} = 2\times10^{-6}$, $\varepsilon_{y0} = 3\times10^{-6}$, $\gamma_{xy0} = 4\times10^{-6}$, $\kappa_x = 1.2\times10^{-4}$ in^{-1}, $\kappa_y = 1.5\times10^{-4}$ in^{-1}, $\kappa_{xy} = 2.6\times10^{-4}$ in^{-1}. Find the global strains at the top, middle and bottom surface of the laminate.

2) Give expressions for the laminate stiffness matrices **A**, **B** and **D** for an isotropic material in terms of its total thickness t, Young modulus E and Poisson's ratio ν.

3) A beam is made of two bonded isotropic strips as shown in the figure below. The two strips are of equal thickness. Find the laminate stiffness matrices **A**, **B** and **D**.

strip 1, E_1, ν_1
strip 2, E_2, ν_2

4) Find the residual thermal stresses at the top of the 60° ply in a [0°/60°/–60°] graphite epoxy laminate subjected to a temperature change of –120°C. Each lamina is 0.127 mm thick. Use E_1 = 181 GPa, E_2 = 10.3 GPa, ν_{12} = 0.28, G_{12} = 7.17 GPa, $\alpha_1 = 0.02\times10^{-6}$ °C^{-1}, $\alpha_2 = 22.5\times10^{-6}$ °C^{-1}.

5) In certain application (for instance, buckling of cylindrical shells), it is convenient to express the in-plane (or membrane) deformations $\boldsymbol{\varepsilon}_0$ and moments **M** as a function of the membrane forces **N** and curvatures $\boldsymbol{\kappa}$ as

$$\begin{Bmatrix} \boldsymbol{\varepsilon}_0 \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}^* & -\mathbf{B}^* \\ \mathbf{B}^{*T} & \mathbf{D}^* \end{bmatrix} \begin{Bmatrix} \mathbf{N} \\ \boldsymbol{\kappa} \end{Bmatrix}$$

Show that $\mathbf{A}^* = \mathbf{A}^{-1}$, $\mathbf{B}^* = \mathbf{A}^{-1}\mathbf{B}$ and $\mathbf{D}^* = \mathbf{D} - \mathbf{B}\mathbf{B}^*$.

6) For the laminate $[\pm\theta]_n$ determine B_{xs} in terms of the total thickness t, n, Q_{11}, Q_{22}, Q_{12}, Q_{66} and the fiber angle θ. All layers have equal thickness.

7) Obtain matrices **A**, **B** and **D** for the laminate [0°/90°] with the following properties: E_1 = 145 GPa, E_2 = 10.5 GPa, G_{12} = 7.5 GPa e ν_{12} = 0.28. Each layer has thickness h = 0.25 mm.

8) Prove that the laminate $\left[\frac{\pi}{n}/\frac{2\pi}{n}/\ldots/\frac{n-1}{n}\pi/\pi\right]$ is isotropic, i.e., matrix **A** is orientation independent. Hint: use lamination parameters and lamina invariants to show that $\sum_{i=1}^{n}\cos\left[2\left(\frac{i\pi}{n}+\theta\right)\right]$, $\sum_{i=1}^{n}\cos\left[4\left(\frac{i\pi}{n}+\theta\right)\right]$, $\sum_{i=1}^{n}\sin\left[2\left(\frac{i\pi}{n}+\theta\right)\right]$ and $\sum_{i=1}^{n}\sin\left[4\left(\frac{i\pi}{n}+\theta\right)\right]$ are independent of θ.

9) Determine thermal forces N_x^T, N_y^T, N_{xy}^T for the laminate $[\pm 45]_s$ AS4/3501-6 with ΔT = 56°C in terms of the lamina thickness h. Use E_1 = 142 GPa, E_2 = 10.3 GPa, G_{12} = 7.2 GPa, ν_{12} = 0.27, $\alpha_1 = -0.9\times10^{-6}$ °C^{-1} e $\alpha_2 = 30.0\times10^{-6}$ °C^{-1}.

10) A lamina is subjected to temperature increase ΔT. Determine the stress σ_x that must be applied in order to avoid shear strain. Find σ_x as a function of ΔT, the fiber angle θ and the mechanical properties E_1, E_2, G_{12}, ν_{12}, α_1, α_2.