

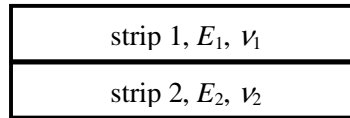
Third assignment of MP-206

Resolution of this assignment is voluntary

1) A laminate of 0.015 in thickness under a complex load gives the following midplane strains and curvatures: $\epsilon_{x0} = 2 \times 10^{-6}$, $\epsilon_{y0} = 3 \times 10^{-6}$, $\gamma_{xy0} = 4 \times 10^{-6}$, $\kappa_x = 1.2 \times 10^{-4} \text{ in}^{-1}$, $\kappa_y = 1.5 \times 10^{-4} \text{ in}^{-1}$, $\kappa_{xy} = 2.6 \times 10^{-4} \text{ in}^{-1}$. Find the global strains at the top, middle and bottom surface of the laminate.

2) Give expressions for the laminate stiffness matrices **A**, **B** and **D** for an isotropic material in terms of its total thickness t , Young modulus E and Poisson's ratio ν .

3) A beam is made of two bonded isotropic strips as shown in the figure below. The two strips are of equal thickness. Find the laminate stiffness matrices **A**, **B** and **D**.



4) Find the residual thermal stresses at the top of the 60° ply in a $[0^\circ/60^\circ/-60^\circ]$ graphite epoxy laminate subjected to a temperature change of -120°C . Each lamina is 0.127 mm thick. Use $E_1 = 181 \text{ GPa}$, $E_2 = 10.3 \text{ GPa}$, $\nu_{12} = 0.28$, $G_{12} = 7.17 \text{ GPa}$, $\alpha_1 = 0.02 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$, $\alpha_2 = 22.5 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$.

5) In certain application (for instance, buckling of cylindrical shells), it is convenient to express the in-plane (or membrane) deformations ϵ_0 and moments **M** as a function of the membrane forces **N** and curvatures κ as

$$\begin{Bmatrix} \epsilon_0 \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}^* & -\mathbf{B}^* \\ \mathbf{B}^{*T} & \mathbf{D}^* \end{bmatrix} \begin{Bmatrix} \mathbf{N} \\ \kappa \end{Bmatrix}$$

Show that $\mathbf{A}^* = \mathbf{A}^{-1}$, $\mathbf{B}^* = \mathbf{A}^{-1}\mathbf{B}$ and $\mathbf{D}^* = \mathbf{D} - \mathbf{B}\mathbf{B}^*$.

6) For the laminate $[\pm\theta]_n$ determine B_{xs} in terms of the total thickness t , n , Q_{11} , Q_{22} , Q_{12} , Q_{66} and the fiber angle θ . All layers have equal thickness.

7) Obtain matrices **A**, **B** and **D** for the laminate $[0^\circ/90^\circ]$ with the following properties: $E_1 = 145 \text{ GPa}$, $E_2 = 10.5 \text{ GPa}$, $G_{12} = 7.5 \text{ GPa}$ e $\nu_{12} = 0.28$. Each layer has thickness $h = 0.25 \text{ mm}$.

8) Prove that the laminate $\left[\frac{\pi}{n} / \frac{2\pi}{n} / \dots / \frac{n-1}{n} \pi / \pi \right]$ is isotropic, i.e., matrix **A** is orientation independent. Hint: use lamination parameters and lamina invariants to show that $\sum_{i=1}^n \cos \left[2 \left(\frac{i\pi}{n} + \theta \right) \right]$, $\sum_{i=1}^n \cos \left[4 \left(\frac{i\pi}{n} + \theta \right) \right]$, $\sum_{i=1}^n \sin \left[2 \left(\frac{i\pi}{n} + \theta \right) \right]$ and $\sum_{i=1}^n \sin \left[4 \left(\frac{i\pi}{n} + \theta \right) \right]$ are independent of θ .

9) Determine thermal forces N_x^T , N_y^T , N_{xy}^T for the laminate $[\pm 45]_s$ AS4/3501-6 with $\Delta T = 56^\circ\text{C}$ in terms of the lamina thickness h . Use $E_1 = 142 \text{ GPa}$, $E_2 = 10.3 \text{ GPa}$, $G_{12} = 7.2 \text{ GPa}$, $\nu_{12} = 0.27$, $\alpha_1 = -0.9 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ e $\alpha_2 = 30.0 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$.

10) A lamina is subjected to temperature increase ΔT . Determine the stress σ_x that must be applied in order to avoid shear strain. Find σ_x as a function of ΔT , the fiber angle θ and the mechanical properties E_1 , E_2 , G_{12} , ν_{12} , α_1 , α_2 .