Composite Materials

Class notes

3. Lamina: micromechanical behavior

General comments

The apparent properties of a lamina were addressed

How can the stiffness and strengths of a composite lamina be varied by changing the proportion of fibers (V_f) and matrix (V_m)?

Properties of a lamina can be experimentally determined or mathematically predicted

Micromechanics $\rightarrow$ prediction

Macromechanics $\rightarrow$ measurement

Micromechanics has serious limitations. For instance, perfect bonding between fibers and matrix is assumed

Basic approaches to micromechanics

(1) Mechanics of materials → simplifying assumptions

(2) Elasticity: (i) bounding principles, (ii) exact solutions, (iii) approximate solutions

Objective: Find $C_{ij} = C_{ij}(E_f, \nu_f, V_f, E_m, \nu_m, V_m)$

$$X_i = X_i(X_{if}, V_f, X_{im}, V_m)$$

3.1 Volume and mass fractions; voids

Volume fractions

$v_{c,f,m}$ = volume of composite, fiber and matrix

$$V_f = \frac{v_f}{v_c} \quad , \quad V_m = \frac{v_m}{v_c}$$

$$v_c = v_f + v_m \quad \Rightarrow \quad V_f + V_m = 1$$

Mass fractions

$w_{c,f,m}$ = mass of composite, fiber and matrix

$$W_f = \frac{w_f}{w_c} \quad , \quad W_m = \frac{w_m}{w_c}$$

$$w_c = w_f + w_m \quad \Rightarrow \quad W_f + W_m = 1$$

Density

$\rho_{c,f,m}$ = density of composite, fiber and matrix

$$w_c = \rho_c v_c \quad , \quad w_f = \rho_f v_f \quad , \quad w_m = \rho_m v_m$$

$$W_f = \frac{w_f}{w_c} = \frac{\rho_f v_f}{\rho_c v_c} = \frac{\rho_f}{\rho_c} V_f \quad , \quad W_m = \frac{w_m}{w_c} = \frac{\rho_m v_m}{\rho_c v_c} = \frac{\rho_m}{\rho_c} V_m$$

$$w_c = w_f + w_m \quad \Rightarrow \quad \rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c} = \rho_f V_f + \rho_m V_m$$

$$v_c = v_f + v_m \quad \Rightarrow \quad \frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}$$

Void content

A decrease of 2% to 10% in the matrix dominated properties generally takes place with every 1% increase in void content

$$V_v = \frac{v_v}{v_c}$$

$$v_c = v_f + v_m + v_v$$

Theoretical density: $\rho_{ct} = w_c / (v_f + v_m)$

Experimental density: $\rho_{ce} = w_c / v_c$

$$v_v = \frac{w_c}{\rho_{ce}} - \frac{w_c}{\rho_{ct}} = \frac{w_c}{\rho_{ce}}\left(\frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}\right) \quad \Rightarrow \quad V_v = \frac{v_v}{v_c} = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}}$$

Example: A glass/epoxy lamina consists of a 70% fiber volume fraction. Use properties of glass and epoxy to determine: (i) the density of the lamina, (ii) mass fractions of the glass and epoxy, (iii) volume of composite lamina if the mass of the lamina is 4 kg, (iv) volume and mass of glass and epoxy in (iii). Use $\rho_f = 2500$ kg/m^3 and $\rho_m = 1200$ kg/m^3.

(i) Density of the lamina: $\rho_c = 2500\times0.7 + 1200\times0.3 = 2110$ kg/m^3

(ii) Mass fractions: $W_f = V_f\rho_f/\rho_c = 0.8294$, $W_m = V_m\rho_m/\rho_c = 0.170694$

(iii) $v_c = w_c/\rho_c = 1.896\times10^{-3}$ m^3

(iv) $v_f = V_f v_c = 1.327\times10^{-3}$ m^3, $v_m = V_m v_c = 0.5688\times10^{-3}$ m^3

$w_f = \rho_f v_f = 3.318$ kg, $w_m = \rho_m v_m = 0.6826$ kg

Exercise: The weight fraction of glass in a glass/epoxy composite is 0.8. If the density of the glass is $\rho_f = 2500$ kg/m^3 and the density of the epoxy is $\rho_m = 1200$ kg/m^3 find: (i) fiber and matrix volume fractions and (ii) density of the composite.

3.2 Elastic moduli

Assumptions

The lamina is: initially stress free, linearly elastic, macroscopically homogeneous, macroscopically orthotropic

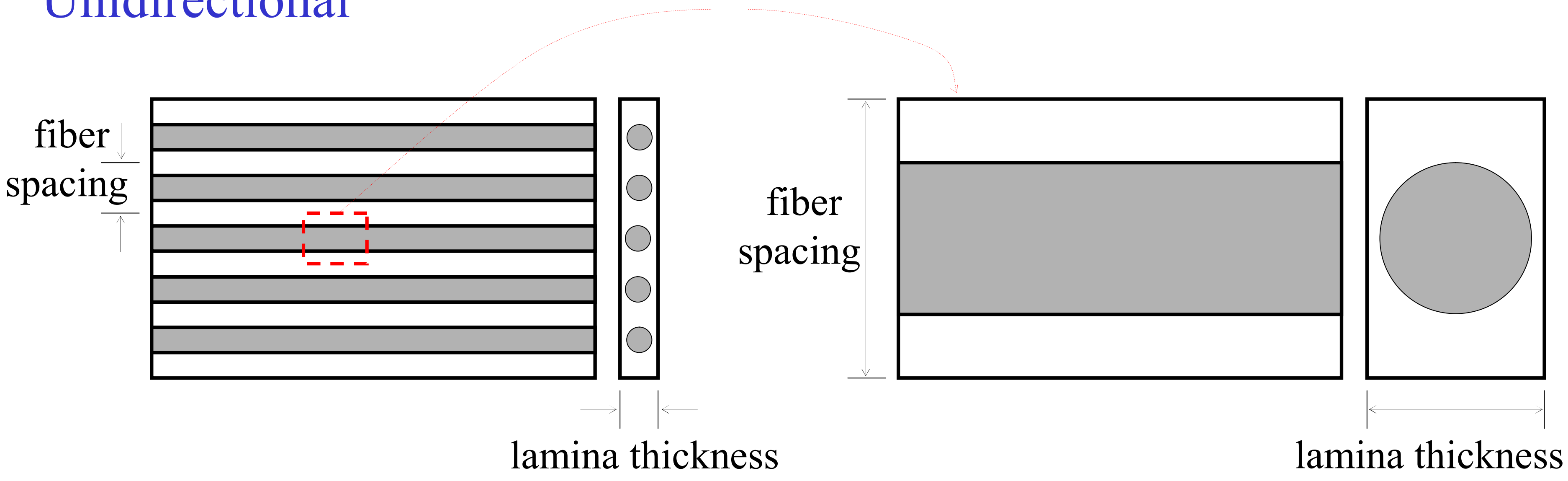
The fibers are: homogeneous, linearly elastic, isotropic, regularly spaced, perfectly aligned, perfectly bonded

The matrix is: homogeneous, linearly elastic, isotropic, void free

Representative volume

Smallest volume where stresses and strains can be regarded as macroscopically uniform and has the correct proportion of fibers and matrix

Unidirectional

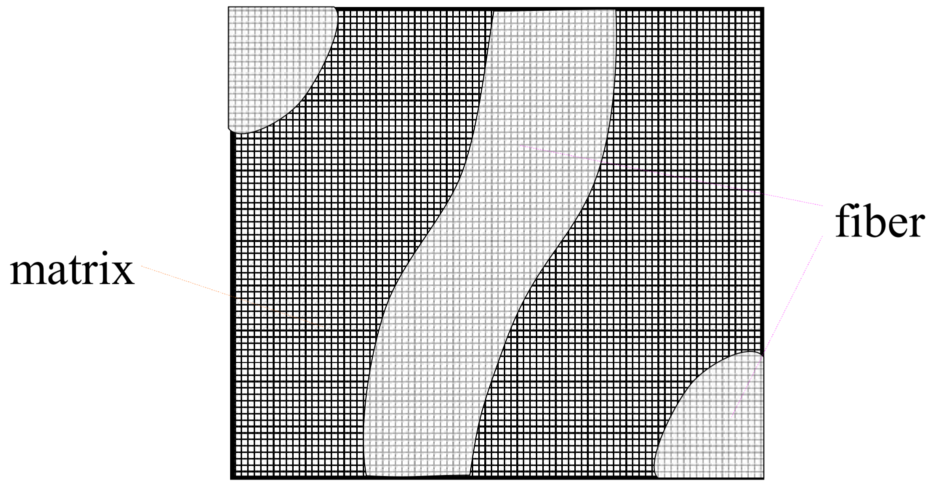


Representative volume

Woven

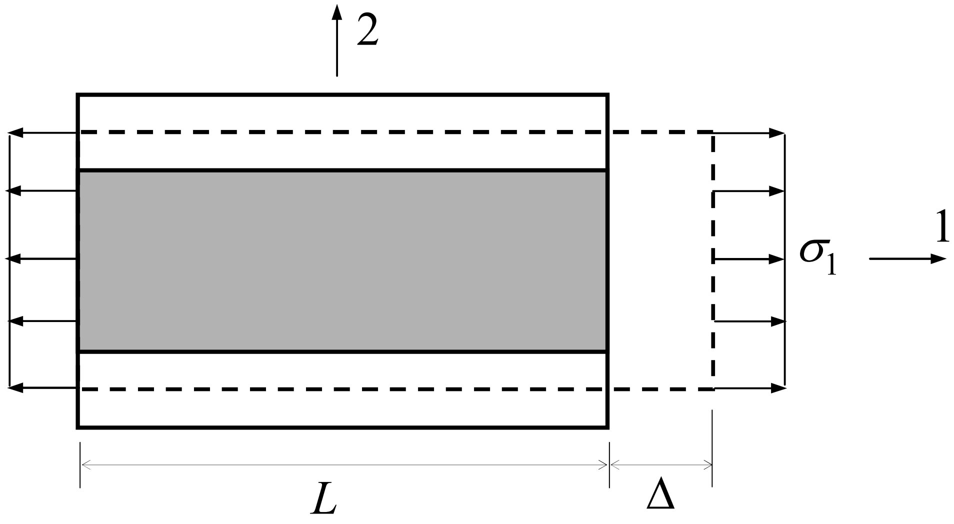
Weave geometry must be considered

FE models are usually used



Mechanics of materials approach

Determination of E_1



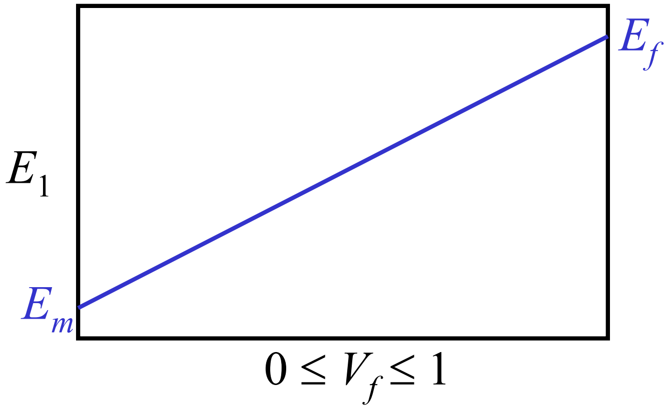
$$\varepsilon_1 = \Delta / L$$

$$\sigma_f = E_f \varepsilon_1$$

$$\sigma_m = E_m \varepsilon_1$$

$$P = \sigma_1 A = \sigma_f A_f + \sigma_m A_m$$

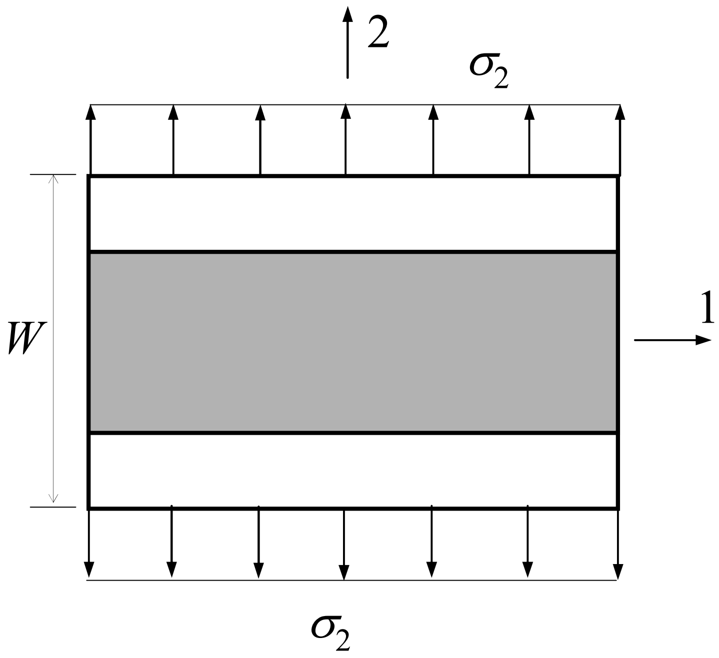
$$E_1 = \frac{P}{A\varepsilon_1} = E_f V_f + E_m V_m$$



Good correlation with E_1 measured experimentally

Mechanics of materials approach

Determination of E_2



$$\varepsilon_f = \sigma_2 / E_f \quad , \quad \varepsilon_m = \sigma_2 / E_m$$

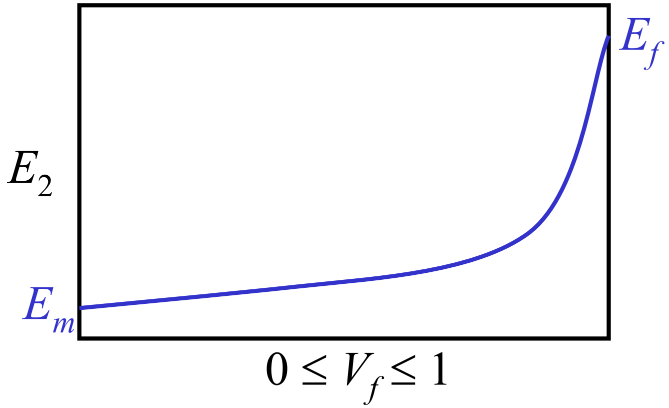
$$\Delta W = \varepsilon_2 W = V_f W \varepsilon_f + V_m W \varepsilon_m$$

$$\sigma_2 = E_2 \varepsilon_2 = E_2 (V_f \varepsilon_f + V_m \varepsilon_m)$$

$$\sigma_2 = E_2 (V_f \sigma_2 / E_f + V_m \sigma_2 / E_m)$$

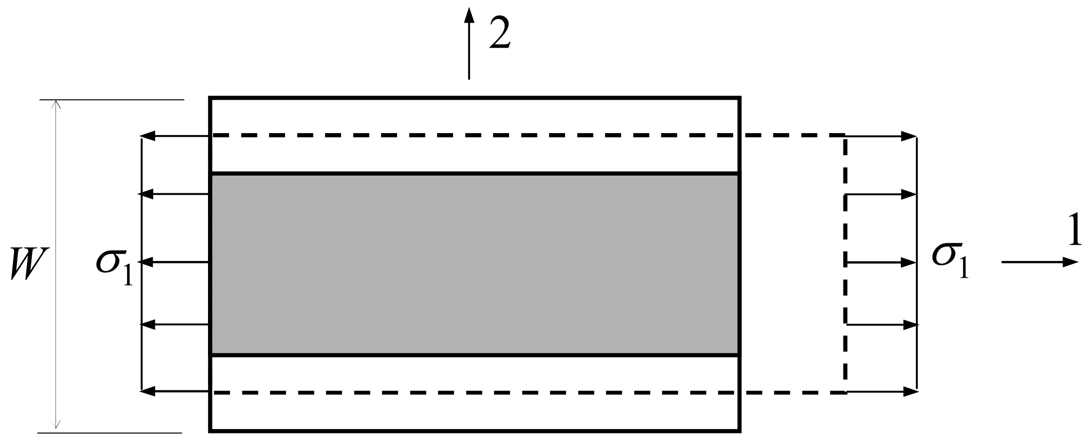
$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

Bad correlation with E_2 measured experimentally



Mechanics of materials approach

Determination of ν_{12}



$$\nu_{12} = -\varepsilon_2 / \varepsilon_1$$

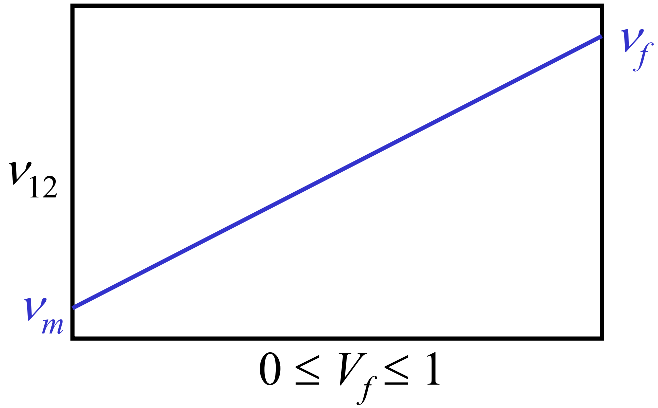
$$\Delta W = -W\varepsilon_2 = W\nu_{12}\varepsilon_1$$

$$\Delta W = \Delta W_f + \Delta W_m$$

$$\Delta W_m = WV_m\nu_m\varepsilon_1$$

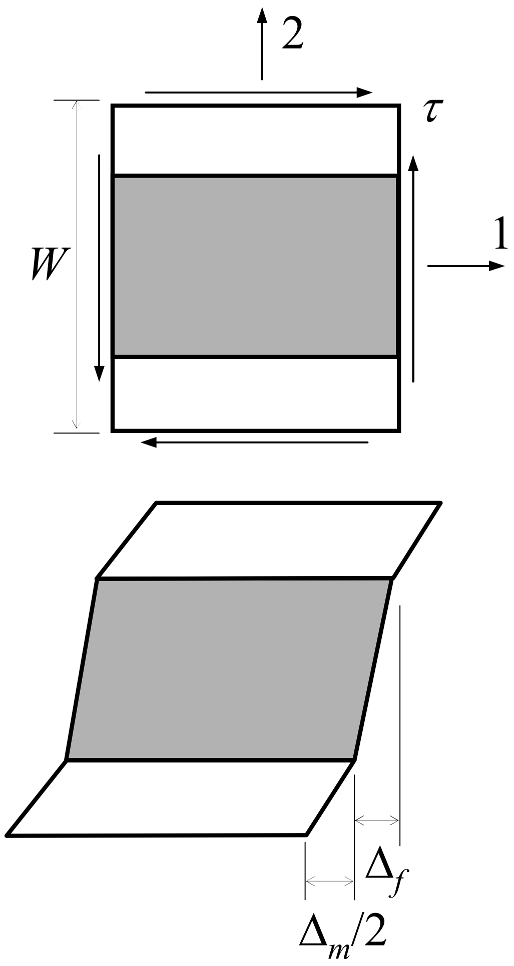
$$\Delta W_f = WV_f\nu_f\varepsilon_1$$

$$\nu_{12} = V_m\nu_m + V_f\nu_f$$



Mechanics of materials approach

Determination of G_{12}



$$\Delta = \gamma W$$

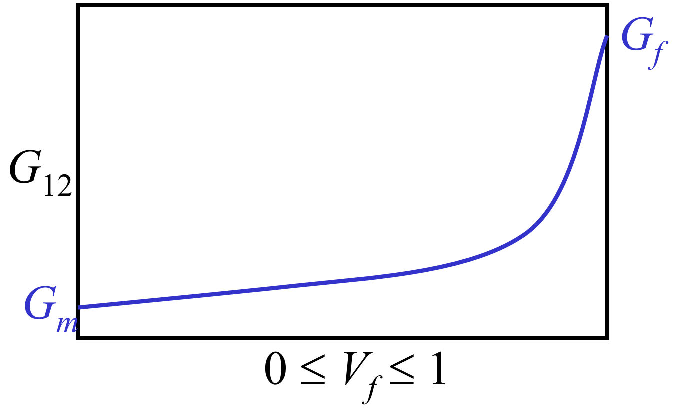
$$\Delta_m = W V_m \gamma_m$$

$$\Delta_f = W V_f \gamma_f$$

$$\Delta = \Delta_m + \Delta_f$$

$$\gamma = \tau / G_{12} \qquad \gamma_m = \tau / G_m \qquad \gamma_f = \tau / G_f$$

$$\frac{1}{G_{12}} = \frac{V_m}{G_m} + \frac{V_f}{G_f}$$



Halpin-Tsai rules of mixture

$$E_1 = E_f V_f + E_m (1 - V_f)$$

$$\nu_{12} = \nu_f V_f + \nu_m (1 - V_f)$$

$$\xi = 1 + 40 V_f^{10}$$

$$\eta_E = \frac{(E_f / E_m) - 1}{(E_f / E_m) + \xi}$$

$$E_2 = E_m \frac{1 + \xi \eta_E V_f}{1 - \eta_E V_f}$$

$$\eta_G = \frac{(G_f / G_m) - 1}{(G_f / G_m) + \xi}$$

$$G_{12} = G_m \frac{1 + \xi \eta_G V_f}{1 - \eta_G V_f}$$

Example: Determine the transverse modulus E_2 of a carbon/epoxy composite with the following properties: $E_f = 85$ GPa, $E_m = 3.4$ GPa, $\nu_f = 0.2$, $\nu_m = 0.3$ and $V_f = 0.7$.

$V_m = 1 - V_f = 0.3$

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \Rightarrow E_2 = E_f E_m / (E_f V_m + E_m V_f) = 10.37 \text{ GPa}$$

$E_2 = 20.71$ GPa (Halpin-Tsai)

$G_f = E_f/(1 + \nu_f)/2 = 35.42$ GPa

$G_m = E_m/(1 + \nu_m)/2 = 1.308$ GPa

$G_{12} = 4.014$ GPa

$G_{12} = 8.131$ GPa (Halpin-Tsai)

Exercise: Determine the transverse modulus E_2 of a silicon carbide/aluminum (SiC/Al) composite with the following properties: $E_f = 366$ GPa, $E_m = 69$ GPa and $V_f = 0.4$.

Exercise: Determine the in-plane shear modulus G_{12} of a glass/epoxy composite with the following properties: $G_f = 28.3$ GPa, $G_m = 1270$ MPa and $V_f = 0.55$.

3.3 Coefficients of thermal expansion

Longitudinal thermal expansion coefficient

$$\sigma_f V_f + \sigma_m V_m = 0$$

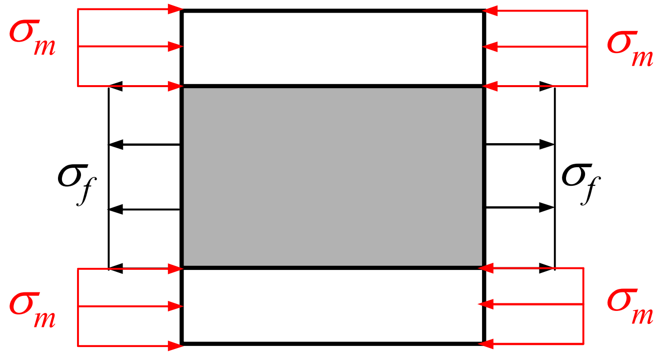
$$\sigma_f = E_f(\varepsilon_f - \alpha_f \Delta T)$$

$$\sigma_m = E_m(\varepsilon_m - \alpha_m \Delta T)$$

$$\varepsilon_f = \varepsilon_m = \varepsilon_1 \quad \Rightarrow \quad \varepsilon_f = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m} \Delta T$$

$$\varepsilon_1 = \alpha_1 \Delta T \quad \Rightarrow \quad \alpha_1 = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m} = \frac{1}{E_1}(\alpha_f E_f V_f + \alpha_m E_m V_m)$$

Transverse thermal expansion coefficient



$$\sigma_f = E_f(\varepsilon_f - \alpha_f \Delta T) =$$
$$E_f(\varepsilon_1 - \alpha_f \Delta T) = E_f(\alpha_1 - \alpha_f)\Delta T$$
$$\sigma_m = E_m(\varepsilon_m - \alpha_m \Delta T) =$$
$$E_m(\varepsilon_1 - \alpha_m \Delta T) = E_m(\alpha_1 - \alpha_m)\Delta T$$

$$\varepsilon_{f2} = \alpha_f \Delta T - \frac{\nu_f \sigma_f}{E_f} \quad , \quad \varepsilon_{m2} = \alpha_m \Delta T - \frac{\nu_m \sigma_m}{E_m}$$

$$\varepsilon_2 = \varepsilon_{f2} V_f + \varepsilon_{m2} V_m \quad \Rightarrow \quad \varepsilon_2 = \left[\alpha_f - \frac{\nu_f E_f(\alpha_1 - \alpha_f)}{E_f}\right] V_f \Delta T + \left[\alpha_m + \frac{\nu_m E_m(\alpha_m - \alpha_1)}{E_m}\right] V_m \Delta T$$

$$\varepsilon_2 = \alpha_2 \Delta T \quad \Rightarrow \quad \alpha_2 = [\alpha_f - \nu_f(\alpha_1 - \alpha_f)] V_f + [\alpha_m + \nu_m(\alpha_m - \alpha_1)] V_m$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad \Rightarrow \quad \alpha_2 = (1 + \nu_f)\alpha_f V_f + (1 + \nu_m)\alpha_m V_m - \alpha_1 \nu_{12}$$

Example: thermal expansion coefficients

$E_f = 85$ GPa,

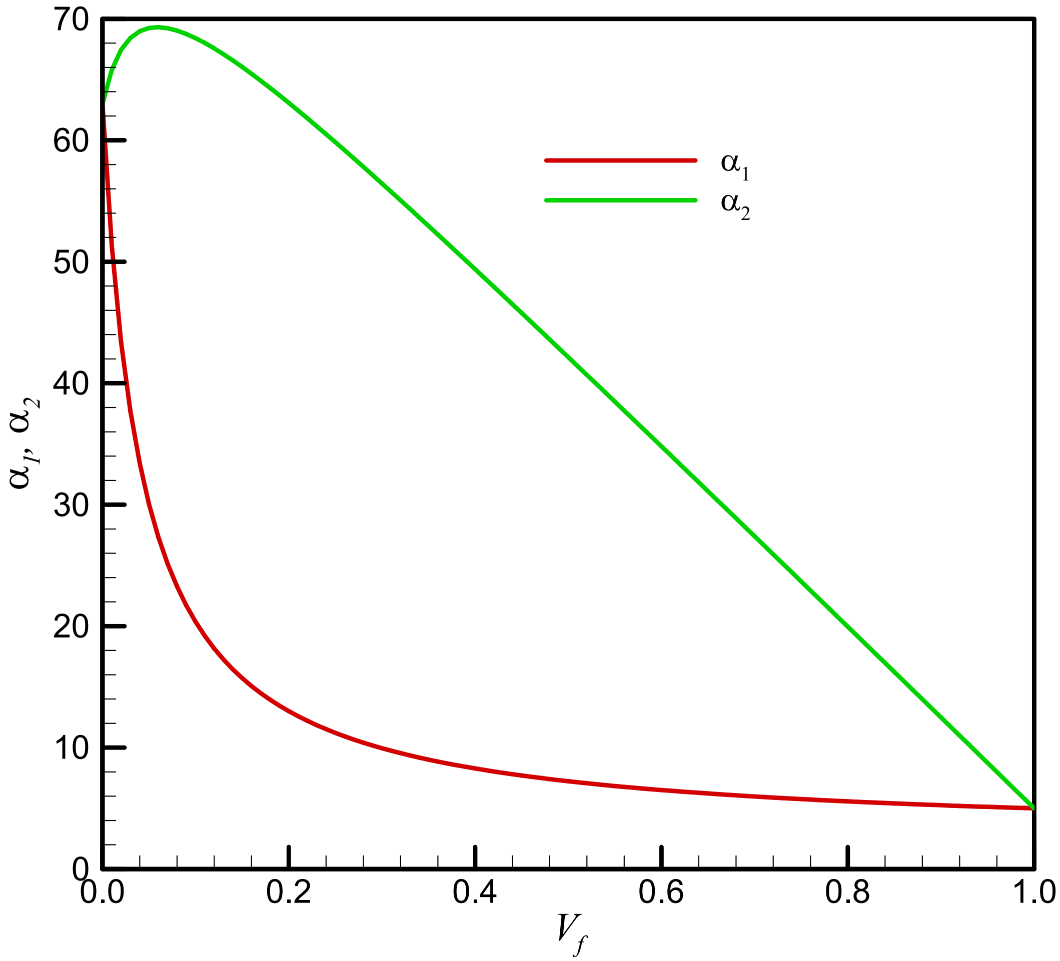
$\nu_f = 0.2$

$\alpha_f = 5\times10^{-6}\ ^{o}C^{-1}$

$E_m = 3.4$ GPa

$\nu_m = 0.3$

$\alpha_m = 63\times10^{-6}\ ^{o}C^{-1}$



Example: Find the coefficients of thermal expansion for a unidirectional glass/epoxy lamina with a 70% fiber volume fraction. Use $E_f = 85$ GPa, $E_m = 3.4$ GPa, $\nu_f = 0.2$, $\nu_m = 0.3$, $\alpha_f = 5.0\times10^{-6}$ $^{o}C^{-1}$ and $\alpha_m = 63.0\times10^{-6}$ $^{o}C^{-1}$.

$$E_1 = E_f V_f + E_m V_m = 60.52 \text{ GPa}, \; \nu_{12} = 0.2300$$

$$\alpha_1 = \frac{\alpha_f E_f V_f + \alpha_m E_m V_m}{E_f V_f + E_m V_m} = \frac{1}{E_1}(\alpha_f E_f V_f + \alpha_m E_m V_m) = 5.978\times10^{-6} \;\; {}^{o}\text{C}^{-1}$$

$$\alpha_2 = (1+\nu_f)\alpha_f V_f + (1+\nu_m)\alpha_m V_m - \alpha_1 \nu_{12} = 27.4\times10^{-6} \;\; {}^{o}\text{C}^{-1}$$

3.4 Ultimate strengths

Preliminaries

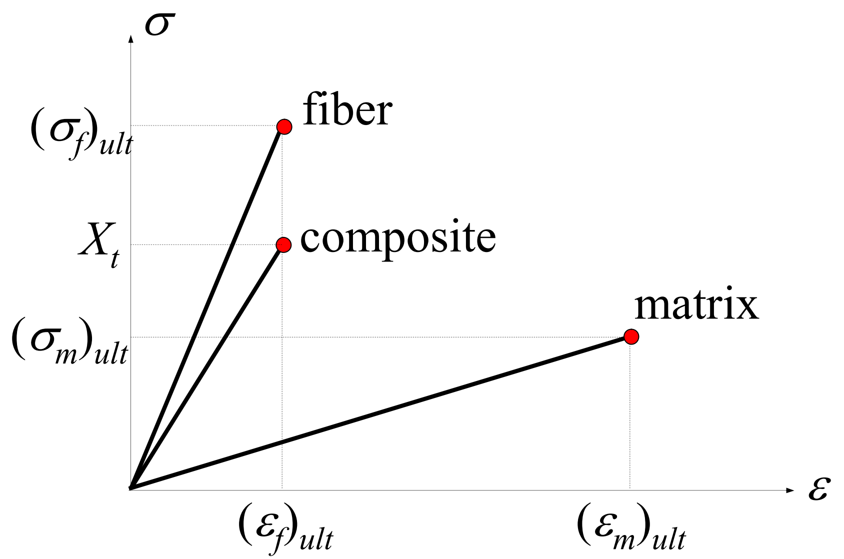
Objective: predict five strength parameters for a unidirectional lamina (X_t = longitudinal tensile, X_c = longitudinal compressive, Y_t = transverse tensile, Y_c = transverse compressive and S = in-plane shear)

Strength parameters are harder to predict than stiffness since strengths are more sensitive to material and geometric nonhomogeneities, fiber-matrix interface, fabrication process and environment

Experimental evaluation of strengths is important

Longitudinal tensile strength

$$(\varepsilon_f)_{ult} = \frac{(\sigma_f)_{ult}}{E_f} \quad , \quad (\varepsilon_m)_{ult} = \frac{(\sigma_m)_{ult}}{E_m}$$



Assumption: when the fibers fail at the strain $(\varepsilon_f)_{ult}$ the whole composite fails

$$X_t = (\sigma_f)_{ult} V_f + (\varepsilon_f)_{ult} E_m (1 - V_f)$$

Question: once the fibers have broken, can the composite take more load? Only if $(\sigma_m)_{ult}(1 - V_f) > X_t$

$$(\sigma_m)_{ult}[1-(V_f)_{\min}] > X_t = (\sigma_f)_{ult}(V_f)_{\min} + (\varepsilon_f)_{ult} E_m [1-(V_f)_{\min}]$$

$$\Rightarrow \quad (V_f)_{\min} < \frac{(\sigma_m)_{ult} - E_m(\varepsilon_f)_{ult}}{(\sigma_f)_{ult} - E_m(\varepsilon_f)_{ult} + (\sigma_m)_{ult}}$$

Example: Find the ultimate tensile strength for a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy $E_f = 85$ GPa, $E_m = 3.4$ GPa, $(\sigma_f)_{ult} = 1550$ MPa, $(\sigma_m)_{ult} = 72$ MPa.

$$(\varepsilon_f)_{ult} = (\sigma_f)_{ult}/E_f = 0.1823\times10^{-1}$$

$$(\varepsilon_m)_{ult} = (\sigma_m)_{ult}/E_m = 0.2771\times10^{-1}$$

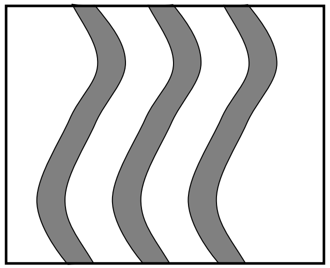
$$X_t = (\sigma_f)_{ult}V_f + (\varepsilon_f)_{ult}E_m(1 - V_f) = 1104 \text{ MPa}$$

Minimum fiber volume: $(V_f)_{\min} = 0.6422\times10^{-2} = 0.6422\%$

Longitudinal compressive strength

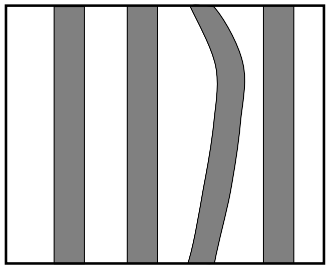
Different failure modes under compressive stress

Microbuckling



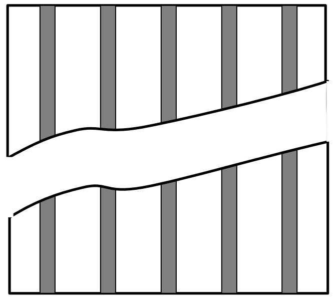
Elasticity models for microbuckling

transverse tensile



$$Y_{\varepsilon t} = (\varepsilon_m^T)_{ult}\left[\frac{d}{s}\left(\frac{E_m}{E_f} - 1\right) + 1\right]$$

shear failure



$$X_c = 2[(\tau_f)_{ult} V_f + (\tau_m)_{ult} V_m]$$

$(\varepsilon_m{}^T)_{ult}$ = ultimate tensile strain of matrix

d = diameter of fibers

s = center-to-center spacing between fibers

Example: Find the longitudinal compressive strength of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy $E_f = 85$ GPa, $E_m = 3.4$ GPa, $(\sigma_f)_{ult} = 1550$ MPa, $(\tau_f)_{ult} = 35$ MPa, $(\sigma_m)_{ult} = 72$ MPa, $(\tau_m)_{ult} = 34$ MPa.

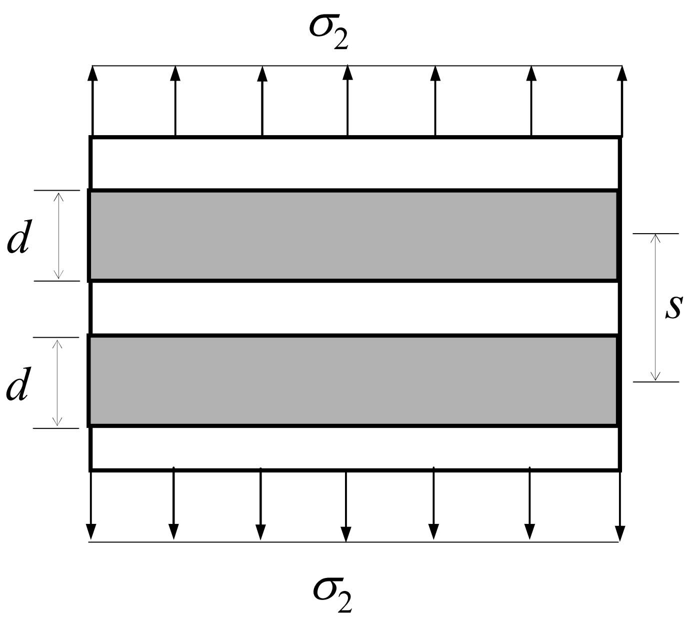
$E_1 = 60.52$ GPa, $\nu_{12} = 0.23$, $d/s = (4\times V_f/\pi)^{1/2} = 0.9441$

$(\varepsilon_m)_{ult} = (\sigma_m)_{ult}/E_m = 0.2117\times10^{-1}$

$X_c = 2[(\tau_f)_{ult}V_f + (\tau_m)_{ult}V_m] = 69.4$ MPa

$$Y_{\varepsilon t} = (\varepsilon_m)_{ult}\left[\frac{d}{s}\left(\frac{E_m}{E_f}-1\right)+1\right] = 0.1983\times10^{-2}$$

Transverse tensile strength



$$\delta_c = s\varepsilon_c \quad , \quad \delta_f = d\varepsilon_f \quad , \quad \delta_m = (s-d)\varepsilon_m$$

$$\delta_c = \delta_f + \delta_m \quad \Rightarrow \quad \varepsilon_c = \frac{d}{s}\varepsilon_f + \left(1 - \frac{d}{s}\right)\varepsilon_m$$

$$E_f\varepsilon_f = E_m\varepsilon_m \quad \Rightarrow \quad \varepsilon_c = \left[\frac{d}{s}\frac{E_m}{E_f} + \left(1 - \frac{d}{s}\right)\right]\varepsilon_m$$

Assumption: transverse failure is due to failure of matrix

$$(\varepsilon_2^T)_{ult} = \left[\frac{d}{s}\frac{E_m}{E_f} + \left(1 - \frac{d}{s}\right)\right](\varepsilon_m^T)_{ult} \quad \Rightarrow \quad Y_t = E_2\left[\frac{d}{s}\frac{E_m}{E_f} + \left(1 - \frac{d}{s}\right)\right](\varepsilon_m^T)_{ult}$$

Example: Find the transverse tensile strength of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy $E_f = 85$ GPa, $E_m = 3.4$ GPa, $(\sigma_f)_{ult} = 1550$ MPa, $(\tau_f)_{ult} = 35$ MPa, $(\sigma_m)_{ult} = 72$ MPa, $(\tau_m)_{ult} = 34$ MPa.

$Y_{\varepsilon t} = 0.1983 \times 10^{-2}$ (from previous example)

$Y_t = E_2 Y_{\varepsilon t} = 20.56$ MPa

Transverse compressive strength

Same equation used to compute transverse tensile strength

$$Y_c = E_2 \left[\frac{d}{s} \frac{E_m}{E_f} + \left(1 - \frac{d}{s} \right) \right] (\varepsilon_m^C)_{ult}$$

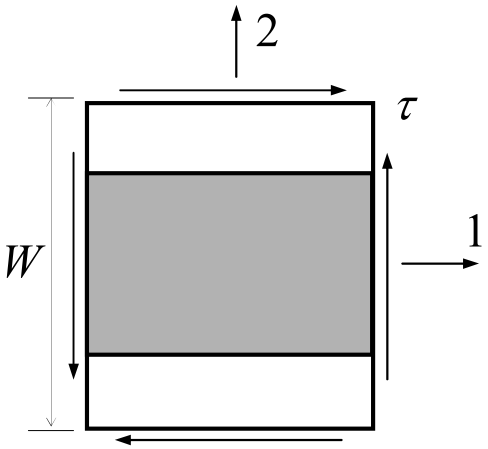
Example: Find the transverse compressive strength of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy $E_f = 85$ GPa, $E_m = 3.4$ GPa, $(\sigma_m^C)_{ult} = 102$ MPa (under compression)

$$d/s = (4\times V_f/\pi)^{1/2} = 0.9441$$

$$(\varepsilon_m^C)_{ult} = (\sigma_m^C)_{ult}/E_m = 0.03$$

$$Y_c = E_2\left[\frac{d}{s}\frac{E_m}{E_f} + \left(1 - \frac{d}{s}\right)\right](\varepsilon_m^C)_{ult} = 29.14\ \text{MPa}$$

Shear strength



$$\Delta_c = s\gamma_c \quad , \quad \Delta_f = d\gamma_f \quad , \quad \Delta_m = (s-d)\gamma_m$$

$$\Delta_c = \Delta_f + \Delta_m \quad \Rightarrow \quad \gamma_c = \frac{d}{s}\gamma_f + \left(1 - \frac{d}{s}\right)\gamma_m$$

$$G_m\gamma_m = G_f\gamma_f \quad \Rightarrow \quad \gamma_c = \left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right]\gamma_m$$

Assumption: shear failure is due to shear failure of matrix

$$\gamma_{ult} = \left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_m)_{ult} \quad \Rightarrow \quad S = G_{12}\left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_m)_{ult}$$

Example: Find the shear strength of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties for glass and epoxy $E_f = 85$ GPa, $E_m = 3.4$ GPa, $\nu_f = 0.2$, $\nu_m = 0.3$, $(\tau_m)_{ult} = 34$ MPa

$$G_f = E_f/(1 + \nu_f)/2 = 35.42 \text{ GPa}$$

$$G_m = E_m/(1 + \nu_m)/2 = 1.308 \text{ GPa}$$

$$G_{12} = G_f G_m/(G_f V_m + G_m V_f) = 4.014 \text{ GPa}$$

$$(\gamma_m)_{ult} = (\tau_m)_{ult}/G_m = 0.2599\times10^{-1}$$

$$S = G_{12}\left[\frac{d}{s}\frac{G_m}{G_f} + \left(1 - \frac{d}{s}\right)\right](\gamma_m)_{ult} = 9.469 \text{ MPa}$$