Analysis and design of composite structures

Class notes
10. Fracture mechanics applied to composite laminates
Basic Principles of Fracture Mechanics

A. A. Griffith: “If a crack is in equilibrium, the decrease of strain energy “U” must be equal to the increase of surface energy “S” due to crack extension “a””

\[ \frac{\partial U}{\partial a} = \frac{\partial S}{\partial a} \]
Basic Principles of Fracture Mechanics

\[
\frac{1}{m} = \frac{P}{e}; \quad \frac{\partial e}{\partial a} = 0
\]

\[
U = \frac{1}{2} Pe \rightarrow \frac{\partial U}{\partial a} = \frac{1}{2} e \frac{\partial P}{\partial a} + \frac{1}{2} P \frac{\partial e}{\partial a} = \frac{1}{2} e \frac{\partial P}{\partial a}
\]
Basic Principles of Fracture Mechanics

\[ \frac{1}{m} = \frac{P}{e}; \quad \frac{\partial e}{\partial a} = 0 \]

\[ U = \frac{1}{2} Pe \rightarrow \frac{\partial U}{\partial a} = \frac{1}{2} e \frac{\partial P}{\partial a} + \frac{1}{2} P \frac{\partial e}{\partial a} = \frac{1}{2} e \frac{\partial P}{\partial a} \]

\[ \frac{\partial P}{\partial a} = \frac{1}{m} \frac{\partial e}{\partial a} + \frac{e}{m} \frac{\partial}{\partial a} \left( \frac{1}{m} \right) = - \frac{P}{m} \frac{\partial m}{\partial a} \]

\[ \frac{\partial U}{\partial a} = - \frac{P^2}{2} \frac{\partial m}{\partial a} \]

\[ G = \frac{\partial U}{\partial A} = - \frac{1}{B} \frac{\partial U}{\partial a} = \frac{P^2}{2B} \frac{\partial m}{\partial a} \]

Strain energy release rate:
General comments

• Generally speaking the failure modes in composite laminates can be divided into two categories:
  ➢ Interlaminar failure modes
  ➢ Intralaminar failure modes
Interlaminar failure modes

- The interlaminar failure modes (also known as “DELAMINATION”) occurs due to the high interlaminar stresses acting on the interface between two adjacent layers.
Interlaminar fracture toughness test methods

DCB-ASTM D 5528-94a

4 ENF - MERL

MMB - ASTM D 6671-01
Mode I delamination – Double Cantilever Beam (DCB)

Total displacement at the beam tip:

$$v = \frac{2Pa^3}{3EI_H} \rightarrow m = \frac{v}{P} = \frac{2a^3}{3EI_H}$$

Mode I strain energy release rate:

$$G_I = \frac{P^2}{2B} \frac{\partial m}{\partial a} = \frac{P^2 a^2}{BEI_H} \rightarrow a = \frac{\sqrt{G_I BEI_H}}{P}$$
Mode I delamination

Combining the tip displacement with the expression for the crack extension we obtain the following relationship between load and tip displacement for delamination propagation regime,

\[ v = \frac{2}{3} \left( \frac{BG_{lc} EI_H}{EI_H P^2} \right)^{3/2} \]

With,

\[ I_F = \frac{Bh^3}{12} \]
\[ I_H = \frac{I_F}{8} \]
Mode I delamination - EXAMPLE

\[ E = 135300 \text{ N/mm}^2 \]
\[ G_{ic} = G_{IIc} = 4.0 \text{ N/mm} \]
\[ a = 30 \text{ mm} \]
\[ h = 3 \text{ mm} \]
\[ B = 1 \text{ mm} \]
Mode II delamination – 3 Point Bending

The derivation of the analytical expressions for mode II delamination require analysis of two different cases:

**Case 1:** Crack length \((a)\) shorter than the half length of the beam \((a < L)\)

**Case 2:** Crack length \((a)\) longer than the half length of the beam \((a > L)\)
Mode II delamination – 3 Point Bending

The derivation of the analytical expressions for mode II will be based on the principle of virtual work: “An elastic body with finite dimensions is on equilibrium when the virtual work done by external forces is equal to virtual strain energy for any arbitrary displacement”

$$\delta W_e = \delta U$$
Mode II delamination – 3 Point Bending

\[ \delta W_e = \delta U \]

\[ W_e = P \nu \rightarrow \delta W_e = \delta P \nu \]

\[ U = \frac{1}{2} \int_{x_1}^{x_2} EI_H \left( \frac{d^2 \nu}{dx^2} \right)^2 dx = \frac{1}{2} \int_{x_1}^{x_2} \frac{M^2}{EI_H} dx \]

\[ \delta U = \int_{x_1}^{x_2} \frac{M \delta M}{EI_H} dx = \int_{x_1}^{x_2} \frac{MM_u}{EI_H} dx \]
Mode II delamination – 3 Point Bending

Case 1: Crack length \((a)\) shorter than the half length of the beam \((a < L)\)

\[
0 \leq x < a_0 \\
M(x) = \frac{P}{4} x \rightarrow M_u(x) = \frac{x}{4} \\
a_0 \leq x < L \\
M(x) = \frac{P}{2} x \rightarrow M_u(x) = \frac{x}{2} \\
L \leq x < 2L \\
M(x) = \frac{P}{2} x - P(x - L) \rightarrow M_u(x) = \frac{x}{2} - (x - L)
\]
Mode II delamination – 3 Point Bending

Displacement in the middle of the beam,

\[ \nu = 2 \int_0^{a_0} \frac{P x^3}{16 E I_H} \, dx + \int_{a_0}^{L} \frac{P x^3}{4 E I_F} \, dx + \int_{L}^{2L} \frac{P (2L-x)^2}{4 E I_F} \, dx \]

\[ \nu = \frac{P (2L^3 + 3a_0^3)}{12 E I_F} \]

The relationship between load and displacement previously defined describes the linear portion (before crack propagation) of the 3 point bending test structural response.
Mode II delamination – 3 Point Bending

The mode II strain energy release rate is given by,

$$G_{II} = \frac{P^2}{2B} \frac{\partial C}{\partial a} = \frac{P^2}{2B} \frac{\partial (\nu P^{-1})}{\partial a} = \frac{3P^2a^2}{8BEI_F}$$

with,

$$C = \frac{\nu}{P} = \frac{(2L^3 + 3a^3)}{12EI_F}$$

The crack length can be expressed as,

$$a = \frac{\sqrt{8G_{II}BEI_F}}{\sqrt{3P}}$$
Mode II delamination – 3 Point Bending

Substituting the expression for \( a \) into the Load (\( P \)) \times \text{Displacement} (\( v \)) relationship results in the following expression for the crack propagation regime,

\[
v = \frac{P}{12EI_F} \left[ 2L^3 + \frac{(8G_{II}BEI_F)^{3/2}}{\sqrt{3P^3}} \right]
\]
Mode II delamination – 3 Point Bending

Case 2: Crack length \((a)\) longer than the half length of the beam \((a > L)\)

\[
0 \leq x < L \quad M(x) = \frac{P}{4} x \rightarrow M_u(x) = \frac{x}{4}
\]

\[
L \leq x < a_0 \quad M(x) = \frac{P}{4} x - \frac{P}{2} (x - L) \rightarrow M_u(x) = \frac{x}{4} - \frac{1}{2} (x - L)
\]

\[
a_0 \leq x < 2L \quad M(x) = \frac{P}{2} x - P (x - L) \rightarrow M_u(x) = \frac{x}{2} - (x - L)
\]
Mode II delamination – 3 Point Bending

Displacement in the middle of the beam,

\[ v = 2 \int_0^L \frac{P x^3}{16EI_H} \, dx + 2 \int_L^a \frac{P (2L - x)^2}{16EI_H} \, dx + \int_a^{2L} \frac{P (2L - x)^2}{4EI_F} \, dx \]

\[ v = \frac{P}{3EI_F} \left( 2L^3 - \frac{3}{4} \left( 2L - a \right)^3 \right) \]
Mode II delamination – 3 Point Bending

The mode II strain energy release rate is given by,

\[ G_{II} = \frac{P^2}{2B} \frac{\partial C}{\partial a} = \frac{P^2}{2B} \frac{\partial (\nu P^{-1})}{\partial a} = \frac{3P^2 (2L - a)^2}{8BEI_F} \]

with,

\[ C = \frac{\nu}{P} = \frac{1}{3EI_F} \left( 2L^3 - \frac{3}{4} (2L - a)^3 \right) \]

The crack length can be expressed as,

\[ a = 2L - \frac{\sqrt{8G_{II} BEI_F}}{\sqrt{3P}} \]
Mode II delamination – 3 Point Bending

Substituting the expression for \( a \) into the Load \( (P) \times \) Displacement \( (\nu) \) relationship results in the following expression for the crack propagation regime,

\[
\nu = \frac{P}{3EI_F} \left[ 2L^3 - \frac{(8G_{II}BEI_F)^3}{4\sqrt{3}P^3} \right]
\]
Mode II delamination - EXAMPLE

\[ E = 135300 \text{ N/mm}^2 \]
\[ G_{lc} = G_{llc} = 4.0 \text{ N/mm} \]
\[ a = 30 \text{ mm} \]
\[ L = 50 \text{ mm} \]
\[ h = 3 \text{ mm} \]
\[ B = 1 \text{ mm} \]
Mixed-mode delamination – Mixed-Mode Bending (MMB)
Mixed-mode delamination – Mixed-Mode Bending (MMB)
Mixed-mode delamination – Mixed-Mode Bending (MMB)

The MMB test superposition analysis results in,

\[ P_I = \left( \frac{3c - L}{4L} \right) P \quad P_{II} = \left( \frac{c + L}{L} \right) P \]

For \( a < L \) the expressions for displacements caused by mode I and mode II loadings are given by,

\[ v_I = \frac{2}{3} \frac{P_I a^3}{EI_H} \quad v_{II} = \frac{P_{II} \left( 2L^3 + 3a^3 \right)}{12EI_F} \]
Mixed-mode delamination – Mixed-Mode Bending (MMB)

The corresponding strain energy release rates are,

\[ G_I = \frac{P_I^2}{2B} \frac{\partial (v_I P_I^{-1})}{\partial a} = \frac{P_I^2 a^2}{BEI_H} \]

\[ G_{II} = \frac{P_{II}^2}{2B} \frac{\partial (v_{II} P_{II}^{-1})}{\partial a} = \frac{3P_{II}^2 a^2}{8BEI_F} \]

The expression for mixed mode ratio is given by,

\[ \frac{G_I}{G_{II}} = \frac{4}{3} \left( \frac{3c - L}{c + L} \right)^2 \]
Mixed-mode delamination – Mixed-Mode Bending (MMB)

The criterion for mixed-mode delamination can be written in terms of the following Power-Law,

\[
\left( \frac{G_I}{G_{IC}} \right)^{\alpha} + \left( \frac{G_{II}}{G_{IIC}} \right)^{\alpha} = 1
\]

where:

\( G_{IC} \rightarrow \) Mode I critical strain energy release rate (obtained from DCB tests)
\( G_{IIC} \rightarrow \) Mode II critical strain energy release rate (obtained from 3PBT tests)
Mixed-mode delamination – Mixed-Mode Bending (MMB)

\[
G_I = \frac{P_I^2 \alpha^2}{BEI_H} \\
G_{II} = \frac{3P_{II}^2 \alpha^2}{8BEI_F}
\]

\[
\left( \frac{G_I}{G_{lc}} \right)^\alpha + \left( \frac{G_{II}}{G_{IIc}} \right)^\alpha = 1
\]

Results in the following expression for the crack length,

\[
\alpha = \left[ \left( \frac{P_I^2}{BEI_H G_{lc}} \right)^\alpha + \left( \frac{3P_{II}^2}{8BEI_F G_{IIc}} \right)^\alpha \right]^{-1/2\alpha}
\]
Mixed-mode delamination – Mixed-Mode Bending (MMB)

\[ a = \left[ \left( \frac{P_I^2}{BEI_H G_{lc}} \right)^{\alpha} + \left( \frac{3P_{II}^2}{8BEI_F G_{llc}} \right)^{\alpha} \right]^{-1/2} \]

The displacements at the tip and the middle of the beam are written in terms of the mixed mode crack length,

\[ v_I = \frac{2}{3} \frac{P_I a^3}{EI_H} \]
\[ v_{II} = \frac{P_{II} \left( 2L^3 + 3a^3 \right)}{12EI_F} \]
Mixed-mode delamination – EXAMPLE

\[ E = 135300 \text{ N/mm}^2 \]
\[ G_{lc} = G_{llc} = 4.0 \text{ N/mm} \]
\[ a = 30 \text{ mm} \]
\[ L = 50 \text{ mm} \]
\[ h = 3 \text{ mm} \]
\[ B = 1 \text{ mm} \]
\[ c = 41.5 \text{ mm} \]
Intralaminar failure modes

- The intralaminar failure modes are related to the local in-plane and out-of-plane stresses combined or acting individually on the layers leading to matrix cracking in tension or compression and/or shear, fibre failure either in compression or tension.
Intralaminar fracture toughness test methods

- ASTM E 399-90
- DEN (Double Edge-Notched) Specimens
- 4PBT (Four point bending test Method)

- All test methods available in the open literature were originally developed for metals
- Modifications in the test method are required to handle material anisotropy, different lay-ups & specimen geometries
Intralaminar fracture toughness test methods

ASTM E 399-90
Intralaminar fracture toughness test methods

ASTM E 399-90

Test usually carried out under displacement control
Intralaminar fracture toughness test methods

ASTM E 399-90

\[ K_I^i = \left( \frac{P_i}{h} \right) (w)^{-0.5} F(a_i) \]

\[ F(a_i) = \frac{2 + a_i / w}{(1 - a_i / w)^{1.5}} \left[ 0.886 + 4.64(a_i / w) - 13.32(a_i / w)^2 + 14.72(a_i / w)^3 - 5.6(a_i / w)^4 \right] \]

\[ G_I^i = \left( K_I^i \right)^2 (2\bar{E}_{11}/\bar{E}_{22})^{-0.5} \left[ (\bar{E}_{11} / \bar{E}_{22})^{0.5} + \bar{E}_{11} / 2\bar{G}_{12} - \sqrt{\nu_{12}} \right]^{0.5} \]
Intralaminar fracture toughness test methods

DEN (Double Edge-Notched) Specimens

Instituto Tecnológico de Aeronáutica

MP-206
Intralaminar fracture toughness test methods

DEN (Double Edge-Notched) Specimens

\[
K_i^i = f\left(\frac{a_i}{w}\right)\sigma F \sqrt{a_i}
\]

\[
F(a_i) = 1.98 + 0.36\left(\frac{2a_i}{w}\right) - 2.12\left(\frac{2a_i}{w}\right)^2 + 3.42\left(\frac{2a_i}{w}\right)^3
\]

\[
G_i^i = \frac{(K_i^i)^2}{E_2}
\]

Test usually carried out under displacement control
Intralaminar fracture toughness test methods

4PBT (Four point bending test Method)
Intralaminar fracture toughness test methods

4PBT (Four point bending test Method)

\[ K_i^i = \frac{6Pc}{2wh^2} \sqrt{\pi a_i} F(a_i) \]

\[ F(a_i) = 1.12 - 1.39(a_i / w) + 7.32(a_i / w)^2 - 13.1(a_i / w)^3 + 14(a_i / w)^4 \]

\[ G_i^i = \frac{(K_i^i)^2 (1 - \nu_{12}^2)}{E_2} \]

Test usually carried out under displacement control
Intralaminar fracture toughness test methods

How to account for material anisotropy?

DERIVATION OF A CONSISTENT $F(a_i)$ FUNCTION:

1- Compute $J$-integral for different crack lengths

2- For each $J_i$ compute new values for $F(a_i)$:

$$F(a_i) = \frac{J_i^{0.5} (\bar{E}_{11} \bar{E}_{22})^{0.25} h \sqrt{w}}{(\alpha / 2 + \beta / 2)^{0.25} P_i} \quad \alpha = \sqrt{\bar{E}_{11} / \bar{E}_{22}} \quad \beta = \bar{E}_{11} / 2\bar{G}_{12} - \nu_{12}$$

3- Plot $F(a)$ vs. crack length and find the function which best fit these points
Intralaminar fracture toughness test methods

ASTM E399-90 overestimates the toughness values quite considerably!